Algebra:
Concepts and Connections

Unit 7:

Investigating Data

Students will explore statistical problems using univariate and bivariate quantitative data and answer statistical, investigative questions using the four-step statistical problem-solving process.

MATHEMATICS

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Unit Overview

Investigating Data

Students will collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems.

Students will represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems. Students will answer statistical questions and solve real-world problems by analyzing and interpreting univariate and bivariate data.

Standards Addressed in this Unit

A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
- **A.MM.1.1:** Explain applicable, mathematical problems using a mathematical model.
- **A.MM.1.2:** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities domains.
- **A.MM.1.4:** Use various mathematical representations and structures with this information to represent and solve real-life problems.

A.DSR.10: Collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems; Represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.

- **A.DSR.10.1:** Use statistics appropriate to the shape of the data distribution to compare center (median and mean) and variability (interquartile range, standard deviation) of two or more distributions by hand and using technology.
- **A.DSR.10.2:** Interpret differences in shape, center, and variability of the distributions in the framework, accounting for possible effects of extreme data points (outliers). *(NOTE: The problem framework should include contextual situations to apply the mathematical concept.)*
- **A.DSR.10.3:** Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
- **A.DSR.10.4:** Interpret the slope (predicted rate of change) and the intercept (constant term) of a linear model in the framework of the data. *(NOTE: The problem framework should include contextual situations to apply the mathematical concept.)*
- **A.DSR.10.5:** Calculate the line of best fit and interpret the correlation coefficient, r, of a linear fit using technology. Use r to describe the strength of the goodness of fit of the

regression. Use the linear function to make predictions and assess how reasonable the prediction is in context.

- **A.DSR.10.6**: Decide which type of function is most appropriate by observing graphed data.
- **A.DSR.10.7:** Distinguish between correlation and causation.

Mathematical Practices

These 8 Mathematical Practices and the overarching Practice Standard are essential to the instruction in this unit.

A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

A.MP.1: Make sense of problems and persevere in solving them.

A.MP.2: Reason abstractly and quantitatively.

A.MP.3: Construct viable arguments and critique the reasoning of others.

A.MP.4: Model with mathematics.

A.MP.5: Use appropriate tools strategically.

A.MP.6: Attend to precision.

A.MP.7: Look for and make use of structure.

A.MP.8: Look for and express regularity in repeated reasoning.

Algebra: Concepts and Connections Comprehensive Course Overview

The Comprehensive Course Overview for Algebra: Concepts and Connections provides additional details and information related to teaching and learning the mathematics standards in the course. To get more information, click on the link to the Algebra Course Overview below.

LINK TO ALGEBRA: CONCEPTS & CONNECTIONS COMPREHENSIVE COURSE LEVEL OVERVIEW

Professional Learning Videos

A series of professional learning videos have been developed by the Georgia Department of Education in partnership with Georgia Public Broadcasting to provide instructional support for classroom teachers and teacher leaders. These videos provide a detailed explanation of the standards and instructional suggestions for the classroom to help all learners master the content at high levels. The professional learning video(s) for the standards address in this unit can be found within the GPB Home Classroom and on the GaDOE webpage (www.gadoe.org/mathematics) and within www.georgiastandards.org.

Progressions

Grade 8 – Advanced Algebra: Concepts & Connections MATHEMATICS: LEARNING PROGRESSIONS				
Key Concepts	Grade 8	Algebra: Concepts & Connections	Geometry: Concepts & Connections	Advanced Algebra: Concepts & Connections
FUNCTIONAL & GRAPHICAL REASONING				
Function Families	• Linear functions • Line of best fit	• Linear functions with function notation • Parent graphs of function families • Exponential functions • Quadratic functions	• Function notation to represent transformations	• Exponential and logarithmic functions • Extension of quadratic functions • Radical functions • Polynomial functions • Fundamental Theorem of Algebra • End behavior of polynomial functions • Rational functions • Factor polynomials
DATA & STATISITCAL REASONING				
Ask Statistical Questions; Investigate, Represent and Analyze Data	• Scatter plots of bivariate data • Patterns of association of bivariate data • Straight lines to represent linear associations • Informally assess linear models and interpret slope and y-intercept	• Univariate and bivariate data • Compare and represent center (median and mean) and variability (interquartile range, standard deviation) or two or more distributions by and using technology. • Represent data on a scatter plot. • Calculate the line of best fit and interpret the correlation coefficient, r, of a linear fit using technology. • Distinguish between correlation and causation.	• Categorical data & two-way frequency tables • Compute and interpret expected values for real-life situations.	• Sample surveys, experiments, observational studies • Primary and secondary data • Ethics, privacy, potential bias, and confounding variables • Z-scores • Empirical rule • Descriptive and inferential statistics • Margin of error, confidence intervals • Normal distribution

Unit Description

Investigating Data

In this unit, students will engage in the 4-step statistical problem-solving process to collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems. Students will represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.

When engaging with the content included in this unit, students should solve problems using the four-step statistical problem-solving process. The Framework for Statistical Reasoning should be used in all learning plans to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.
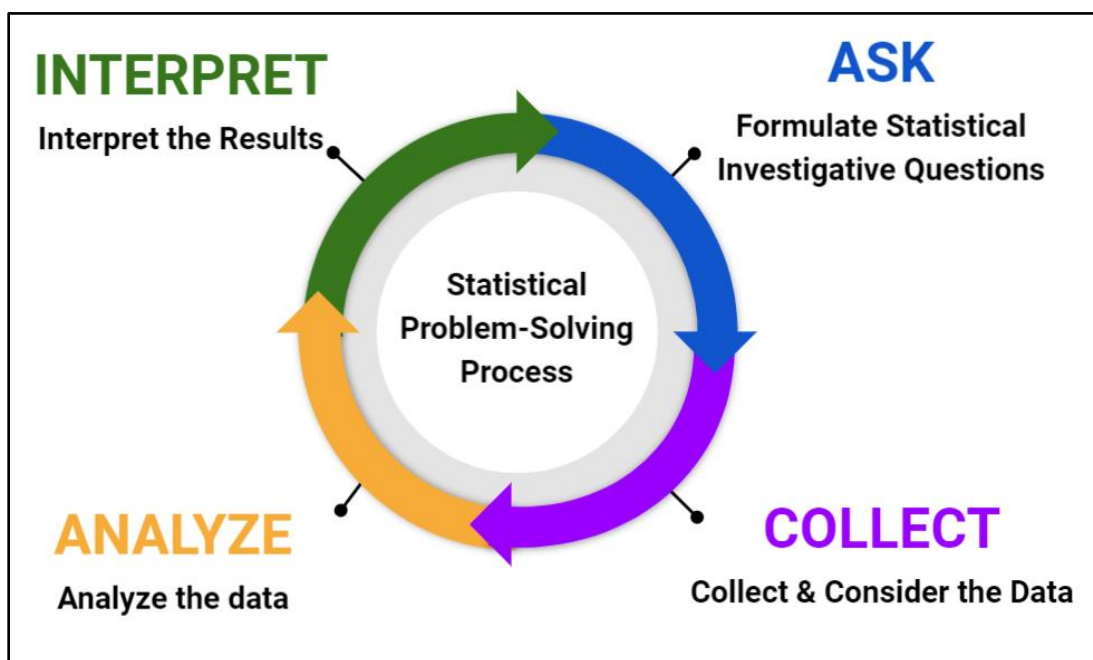

FIGURE 1: GEORGIA FRAMEWORK FOR STATISTICAL REASONING

This unit builds upon students' prior experiences with data, providing students with opportunities to analyze and interpret real-world situations and experiences by using various mathematical representations. Distinctions will be made between univariate and bivariate variables as students identify the data needed to answer statistical investigative questions. Students will revisit and learn techniques for representing and analyzing quantitative data by hand and using technology. Students will use the shapes of distributions to determine appropriate summary statistics and to compare groups of interest. Students will explore patterns of association between two quantitative variables and model data with linear functions where appropriate. An emphasis on making predictions and interpreting model parameters will allow students to connect data to real-world situations.

Evidence-Based Practices

What evidenced-based instructional practices could be employed to support the learner's growth and development in this unit?

For a more detailed description for each practice, please read the [Algebra Guide for Effective Mathematics Instruction](#).

Evidence-Based Practice	Description	Additional Information (Graphic, Chart, Link)
21st Century Learning	The 21st century technology tools can be used to foster reasoning and sense-making with mathematical tasks.	
3-Act Math Tasks	A 3-Act Task is a mathematics task consisting of three distinct parts: an engaging and perplexing Act One, an information- and solution-seeking Act Two, and a solution discussion and a solution-revealing Act Three.	[Effective Instructional Practices Guide](#)
Collaborative Group work	Collaborative group work is encouraged in classrooms to help students make sense of mathematics and engage in productive classroom discourse in groups with their peers.	
Gamification	This instructional strategy finds a balance between games, which are considered 'fun' and voluntary, and learning. Gamification research reveals, however, that the components of well-designed games (goals, rules, feedback, autonomy, and voluntary participation) parallel those of well-designed learning experiences.	[Learning and Games Unpacking the potential of educational gaming: A new tool for gaming research](#) [Measuring Problem Solving Skills via Stealth Assessment in an Engaging Video Game](#) [Scaffolding Game-Based Learning: Impact on Learning Achievements, Perceived Learning, and Game Experiences](#)
Incorporating the 8 Mathematical Practices	The 8 Mathematical Practices are the critical overarching behaviors for each grade level. These eight practices should be incorporated in all lessons and activities for students.	

Modeling with Mathematics	Mathematical modeling is defined as the process of designing and revising representations to solve problems. This process is foundational for students to gain a deep understanding of mathematical concepts in context of real-life phenomena. For students to effectively model with mathematics, the problems must be presented in context. Students make sense of the problem, make decisions about what information is most important, create models using that information, perform operations using those models, interpret the results and validate conclusions. Mathematics is the language of the universe. Mathematics is everywhere. Therefore, this evidence-based practice requires for students to be equipped with the skills and knowledge necessary to use mathematics as a model to explain life around them, which is one of the ultimate purposes of mathematics.	Teaching Channel – Model with Mathematics Mathematical Modeling Missing the Promise of Mathematical Modeling (NCTM) Mathematical Modeling Framework Mathematical Modeling Continuum What is Mathematical Modeling?
Multiple Representations	Mathematical representations are visible or tangible productions – such as diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, physical models, mathematical expressions, formulas and equations, or graphical depictions of mathematical ideas or relationships.	
Number Talks	A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation.	Number Talks Video Effective Instructional Practices Guide
Numeracy Intervention Resources	The use of formative assessments and intervention strategies will help students develop a strong knowledge of foundational skills and concepts in numeracy. The Georgia Numeracy Project offers specific intervention resources to support student learning and development of a solid foundation in numeracy.	Georgia Early Numeracy Project Overview Georgia Secondary Numeracy Project Overview

Patient Problem Solving	Patient problem solving deepens a learner's mathematical literacy. Mathematical literacy means having the ability to problem-solve, reason and analyze information. It is the ability to understand the "language" of mathematics, use numbers to help solve real-world problems, recognize patterns, and generalize pattern recognition to various contexts.	
Pattern Talks	Mathematics is everywhere in the world; therefore, classroom lessons and activities should help students see mathematics in the world and think about the patterns they see. It strengthens their learning and understanding.	Visual Patterns YouCubed Patterns Video
Positive Mathematical Mindsets and Productive Efforts of Learning	Promoting positive mathematical mindsets and productive efforts of learning gives students the time and space needed to work through mathematics problems.	Positive Mathematical Mindsets
Problem-Based Learning	Problem-based learning provides a structure for discovery that helps students internalize learning and leads to greater comprehension. Problem-based learning is student centered and develops both problem-solving strategies and content knowledge aimed at using the power of authentic problem solving to engage students.	"Problem-Based Learning" "New, Strong Evidence for Problem-Based Learning" Video-" Solving Real-World Issues Through Problem-Based Learning"
Project-Based Learning	Project-based learning is an authentic instructional model or strategy in which students plan, implement, and evaluate projects that are real-world applications beyond the classroom.	"What is Project-based learning?": PBL Works "Project-Based Learning Engages K-12 Students with Real-World Challenges" "Project-Based Learning Research Review: Evidence-Based Components of Success" (Edutopia) 21st Century Project Based Learning Video-" What Makes Project-Based Learning a Success?" Video-"Project-Based Learning: Explained"

Standards-Based Grading and Intervention Models	Interventions based on evidence-based practices and high leverage practices that include standards-based grading and intervention models should be incorporated in all classrooms K-12 to support increased student learning of the standards for each grade level.	
Visual Mathematics	Research conducted by Dr. Jo Boaler indicates that the human brains think about mathematics visually. Students who see mathematics visually understand the concepts more deeply.	Brains Think Math Visually Video
Which One Doesn't Belong?	In the Which One Doesn't Belong (WODB) activities, students are presented with four figures and asked to identify which one of the four figures does not belong with the other three figures. The focus is not on finding a singular correct answer, but rather, justifying the selection of their choice.	Which One Doesn't Belong?

Strategies for Teaching and Learning

Some strategies for teaching and learning to help students master the concepts presented in this unit are identified below.

Instructional Strategies	Strategy for Implementation
Higher Order Questioning	Higher order questioning occurs throughout the entire lesson. Teachers should engage students through questioning to assess their level of understanding. This will act as a part of the formative assessment throughout the lesson allowing the teacher to make adjustments to the lesson as they progress.
Engaging/ Activating Strategy	Gets students actively thinking or making a connection with the material being presented that day. The activating strategy is apparent through the engage activity and may also occur in the explore activity.
Academic Discussions	Students should engage in academic discussions throughout the lesson. They should be the driver of the discussion based on the higher order questioning that occurs. Students should be questioning each other and justifying their solutions.
3-Read Protocol	The 3-Read protocol should be used to ensure students are comprehending the situations presented in the learning activities that are task driven. 1) The first read provides the ability for general understanding. Generally, what is the situation about? 2) The second read provides the ability to determine the quantities. What are the needed quantities that are provided? 3) The third read provides the what. What mathematical questions can we ask or what needs to be answered?
Differentiation	Activities are included according to students' unique learning needs providing students with higher academic capabilities the ability to be stretched and those who are struggling to get the appropriate support they may need to access the material.
Summary/Reflection	Teachers should summarize each of the lesson activities prior to dismissal which will allow them the ability to assess where their students are in the learning and provide students a time to reflect. A reflection activity is included for student use.
Graphic Organizer	The use of a graphic organizer allows students to organize their thinking. It allows them to visually categorize new information or review old information.

Instructional Design

Instructional lessons included in this course follow the instructional design involving four sections: Engage, Explore, Apply, and Reflect. Specific descriptions for each section are included below.

- **Engage -** Within this section, the learning experiences include evidence-based instructional strategies that can be used as an introduction that mentally engages students to capture their interest, provides an opportunity to communicate what they know, and allows them to connect what they know to new ideas.
- **Explore -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to engage in hands-on activities to explore the new concept/big idea at a deep level.
- **Apply -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to apply what they have learned in a new situation to develop a deeper understanding of the big idea.
- **Reflect -** Within this section, the learning experiences include evidence-based instructional strategies that allow students the opportunity to review and reflect on their own learning and new understandings.
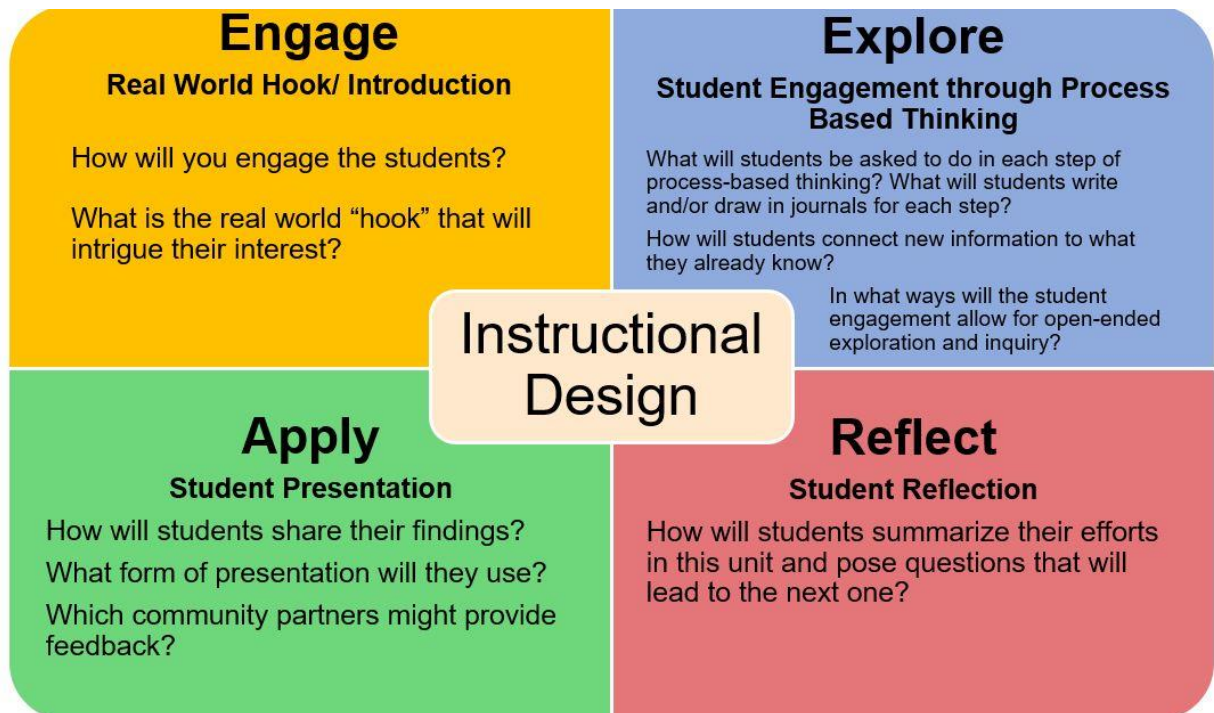
Relevant Academic Vocabulary

Relevant academic vocabulary terms important to the content of this unit are included in the table below. An interactive K-12 Mathematics Glossary is now available as a reference tool for all mathematics teachers.

NOTE: This list is not intended as a vocabulary list for students, but as a reference for teachers that may be used to ensure precise language is used and encouraged by all.

association	linear model
bivariate data	linear fit
box plot	marginal frequencies
categorical values	Mean Absolute Deviation (MAD)
causation	prediction
center	outlier
conditional frequencies	quantitative data
constant	quantitative variables
correlation	range
correlation coefficient	rate of change
data distribution	regression
deviation	scatter plot
dot plot	Second Quartile (Q_2)
First Quartile (Q_1)	shape of data distribution
five-number summary	Shape (Symmetry, Number of Peaks, Direction of Skew, Uniformity)
frequency table (two-way table)	slope
histogram	standard deviation
intercept	third quartile
interquartile range	trend
joint frequencies	univariate data
line of best fit	variability

Concepts and Skills to Maintain

In the early years, students are building foundational knowledge by acquiring a conceptual understanding of categorical data up to 20 objects and eventually numerical data. This foundational knowledge is essential and will be applied to the statistical concepts explored in the secondary years. Students must develop a solid foundation in statistics to be able to ask statistical questions, investigate, and analyze data.

In order for students to be successful, the following skills and concepts need to be maintained:

- Know how to compute the mean, median, interquartile range, mean absolute deviation, and standard deviation by hand in simple cases and using technology with larger data sets.
- Find the lower extreme (minimum), upper extreme (maximum), and quartiles.
- Create a graphical representation of a data set.
- Plot data on a coordinate grid and graph linear functions.
- Recognize characteristics of linear and exponential functions.
- Write an equation of a line given two points.
- Graph data in a scatter plot and determine a trend.
- Determine the slope of a line from any representation.
- Identify the y-intercept from any representation.
- Be able to use graphing technology.
- Understand the meaning of correlation.

Instructional Learning Plans

What learning activities can be used to teach the standards associated with this unit?

Instructional Learning Plan*	Duration	Learning Tasks	Standards/Learning Expectations
The Basketball Star **Teacher Guidance** **Student Reproducibles** **Blackline Masters**	**2 – 3 Days**	- Who is the Better Basketball Player? - Analyzing the Data Part 1 & 2 - Describing Variability Part 1 & 2 - Making a Basket	**A.DSR.10** - A.DSR.10.1 - A.DSR.10.2 **A.MM.1** - A.MM.1.1 - A.MM.1.4 **A.MP**
Learning Plan Description: In this learning plan, students will focus on comparing univariate data in different contexts. Students will analyze data that has already been collected and interpret the results. Once students understand different ways to compare data, they will seek to create their own statistical questions, collect their own data, and analyze and interpret the results.			
Variation in Math Classes **Teacher Guidance** **Student Reproducibles** **Blackline Masters**	**2 – 3 Days**	- Which One Doesn't Belong? - Variation in Math Classes Exploration - An Outlier Rule - Formative Assessment - Reflection	**A.DSR.10** - A.DSR.10.1 - A.DSR.10.2 **A.MM.1** - A.MM.1.1 - A.MM.1.4 **A.MP**
Learning Plan Description: In this learning plan, students will summarize, represent, and interpret data from univariate data. Students will be able to compare data sets from different mathematics classes, collect their own data, and compare those data sets.			
The Strength of Relationships **Teacher Guidance** **Student Reproducibles** **Blackline Masters**	**2 – 3 Days**	- How Do They Relate? - Scatterplot Card Sort - Class Discussion - Investigating Correlations & Making Predictions	**A.DSR.10** - A.DSR.10.3 - A.DSR.10.5 **A.MM.1** - A.MM.1.1 - A.MM.1.4 - A.MM.1.4 **A.MP**
Learning Plan Description: In this learning plan, students will explore scatter plots to determine the strength and direction of the fit and correlation in association with the correlation coefficient. Students will analyze bivariate data to answer statistical questions and solve real-life problems.			

Classroom Mathematics Award **Teacher Guidance** **Student Reproducibles**	**1 – 2 Days**	• Which One Doesn't Belong? • Establishing a Classroom Mathematics Award • Who is the Winner? • Two Stars and a Wish	**A.DSR.10** • A.DSR.10.1 • A.DSR.10.2 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students will use statistical reasoning to compare two distributions and make decision about a classroom mathematics award. Students will calculate measures of spread and use that information to make decisions.			
If the Shoe Fits! **Teacher Guidance** **Student Reproducibles**	**1 – 2 Days**	• Formulate Question(s) • Design and Implement a Data Collection Plan • Exploration • Comparative Box Plots and Scatter Plot Interpretation • Journal Reflection	**A.DSR.10** • A.DSR.10.1 • A.DSR.10.2 • A.DSR.10.3 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students explore and use hypothetical data collected on student shoe print lengths, height, and age in order to help develop a tentative description of a person who entered a school's grounds over a weekend without permission.			
Spaghetti Regression **Teacher Guidance** **Student Reproducibles** **Blackline Masters**	**1 – 2 Days**	• Which One Doesn't Belong? • Spaghetti Regression Exploration • Line of Best Fit • Journal Reflection	**A.DSR.10** • A.DSR.10.3 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students will create scatter plots to develop an understanding of the relationships of bivariate data. With this learning plan, students will investigate the concept of the "goodness-of-fit" and its significance in determining the regression line or best fit line for the data. This includes studying correlations and creating models from which they will predict and make critical judgments.			

Sports Analysis **Teacher Guidance** **Student Reproducibles**	**1 – 2 Days**	• See, Think, Wonder • Equal Salaries for Equal Work? • Exit Ticket	**A.DSR.10** • A.DSR.10.3 • A.DSR.10.5 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students will collect, analyze and interpret the strength of a correlation, describe how two variables are related, and fit a linear function for a scatter plot that suggests a linear association.			
Equal Salaries for Equal Work **Teacher Guidance** **Student Reproducibles**	**1 – 2 Days**	• See, Think, Wonder • Equal Salaries for Equal Work? • Exit Ticket	**A.DSR.10** • A.DSR.10.3 • A.DSR.10.4 • A.DSR.10.5 • A.DSR.10.6 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students will compare additive and multiplicative growth (represented by linear and exponential models) to make predictions and solve problems within the context of salary differences. In this task, students will analyze data sets, create scatter plots, determine the most appropriate mathematical model, and justify their model selection.			
Test Scores and Watching Television **Teacher Guidance** **Student Reproducibles**	**1 – 2 Days**	• Silly Sock Day • Line of Best Fit Exploration • TV/Test Grades • Reflection: Flip	**A.DSR.10** • A.DSR.10.3 • A.DSR.10.4 • A.DSR.10.5 • A.DSR.10.6 • A.DSR.10.7 **A.MM.1** **A.MP**
Learning Plan Description: In this learning plan, students will represent data on a scatter plot, describe how two variables are related, and fit a linear function for a scatter plot that suggests a linear association.			

Numeracy Development Intervention Supports

Georgia Numeracy Project Resources

Georgia Secondary Numeracy Project

The Georgia Secondary Numeracy Project is a numeracy development resource provided by the Georgia Department of Education, which introduces teachers and teacher leaders to the trajectory by which learners acquire and build a solid foundation in numeracy which ultimately supports algebra readiness.

The Georgia Secondary Numeracy Project consists of a two-part universal screener:

- Part 1 of the universal screener is the Diagnostic Assessment. This assessment assesses students' strategy proficiency across 3 domains, Addition and subtraction, multiplication and division, and proportions and ratios. Building upon the strategy stages from the Georgia Early Numeracy Project, the Georgia Secondary Numeracy Project follows this progression.
- The second component of the universal screener is the Written Assessment. Using the Overall Strategy Stage from the Diagnostic Interview, students begin the Written Assessment on the Part equivalent to the Overall Strategy Stage from the Diagnostic Interview. The domains assessed on the Written Assessment are: Relational & Functional Reasoning, Patterning & Algebraic Reasoning, Statistical & Probability Reasoning and Geometric, Spatial & Measurement Reasoning.

In alignment with Georgia's Tiered Supports for Students, the Georgia Secondary Numeracy Project provides tools to target specific skills and provide tiered supports and interventions. To address the identified skills, the Georgia Secondary Numeracy provides numeracy development intervention tasks and activities. Within each unit in the grade level/course, intervention tables are provided to identify specific tasks and activities aligned to the standards and learning objectives discussed within the unit.

The Georgia Secondary Numeracy Project is the perfect complement to Georgia's K-12 Mathematics Standards, aligning with the multiple big ideas and aiding in the development of mathematical reasoning. The Georgia Secondary Numeracy Project supports the various mathematical concepts in Grade 8 through Advanced Algebra: Concepts & Connections.

College and Career Readiness

The GaDOE Roadmap for Reimaging K-12 Education provides teachers and leaders the opportunity to reimagine the learning experiences for students to ensure they are prepared for the future. The goal is to match their passions, their interests, and their abilities with the opportunities they are provided in school and beyond. All students have a uniqueness that can be nurtured and fostered, which can grow into something that is amazing. The goal of the Georgia Department of Education is to ensure that all students have any door of opportunity they desire open for them. Students begin exploring careers and preparing for post-secondary opportunities as early as Kindergarten. As students matriculate through elementary school and middle school, their exploratory opportunities for careers become more refined and focused to assist them with decision-making related to course taking options at the secondary level. Helping students refine their college and career interests, coupled with their strengths and goals will assist them with a great start in identifying post-secondary opportunities. Also, specific course pathways can be selected to ensure they build the foundational knowledge necessary for their college and career pathway of interest.

Providing opportunities for internships and collaboration with local business and community partners will also help students make connections between what they are learning in mathematics class with careers that use the mathematics can be very powerful and eye-opening for the learners. These opportunities should be provided to help students on their quest to become prepared for their future, they are being prepared for life.

At the secondary level, students have the flexibility to participate in personalized mathematics pathways that allow them to deeply explore advanced mathematics content in data science, statistics, quantitative reasoning, and/or calculus based on their future goals and interests. As students participate in advanced-level coursework in high school through the personalized pathways, opportunities should be provided for them to see the connections and experience the mathematics content in the real-world.

Interdisciplinary Teaching and Learning Support

Process-Based Thinking

Process thinking is an important element of interdisciplinary teaching and learning. Process thinking has six components:

1. **Intellectual Challenge and Accomplishment** – this component focuses on developing students' capacity to learn deeply, think critically, and strive for excellence.

2. **Authenticity** – this component focuses on students working on projects that are meaningful and relevant to their community, their lives, and their future.

3. **Public Product** – students' work is publicly displayed, discussed, and critiqued.

4. **Collaboration** – this component focuses on building students' capacity to collaborate with other students in person or online and/or receive guidance from adult mentors and experts.

5. **Project Management** – students use a project management process that enables them to proceed effectively from project initiation to completion.

6. **Reflection** – students reflect on their work and their learning throughout the project.

The six components of process-based thinking connect nicely to the 8 Mathematical Practices that are embedded in all instructional units. These are a part of the Essential Instructional Guidance included for all grade levels and high school courses. Grade 8 lessons include opportunities for students to engage in process-based thinking through exploration and sense-making of the mathematics content standards.

Mathematical Modeling Overview

Mathematical modeling can be explored through interdisciplinary teaching and learning. One instructional model for interdisciplinary teaching and learning is project-based learning. There are six powerful project-based instructional practices:

1. Plan Authentic, Intellectually Demanding Project-Based Learning Units Where Students Master Significant Content and Skills

2. Utilize Sustained, In-Depth Inquiry

3. Engage Students in a Collaborative Problem-Solving/Design Process

4. Foster a Classroom Environment That Supports Student Ownership of Learning

5. Engage in Ongoing and Purposeful Feedback, Revision, and Reflection

6. Include Community Partners in Project Planning, Implementation, and Reflection

Mathematical Modeling Framework

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create. The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.
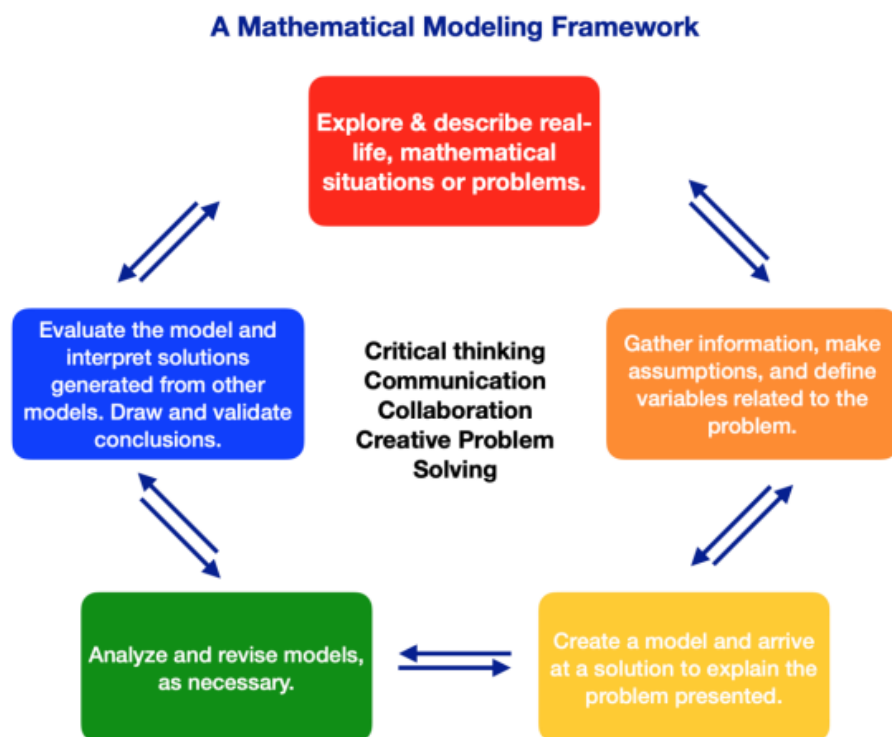
A Mathematical Modeling Framework

Explore & describe real-life, mathematical situations or problems.

Gather information, make assumptions, and define variables related to the problem.

Critical thinking Communication Collaboration Creative Problem Solving

Evaluate the model and interpret solutions generated from other models. Draw and validate conclusions.

Create a model and arrive at a solution to explain the problem presented.

Analyze and revise models, as necessary.

Image adapted from: Suh, Matson, Seshaiyer, 2017

The Mathematical Modeling Framework offers some insight into what modeling with mathematics looks like when implemented in K-12 classrooms.

- Real-life, mathematical situations or problems are investigated.
- Students gather information, make assumptions, and define unknowns (variables).
- Mathematical models are created and used to arrive at a solution to explain the real-life, mathematical situation or problem.
- Models are analyzed and revised as needed.
- Models are evaluated by students. Solutions using different models are interpreted and conclusions are drawn and validated.

When choosing mathematical modeling tasks and activities, keep in mind that these tasks should (1) be interesting and/or important for students to experience, (2) exemplify specific components of the modeling cycle, (3) be doable by real students in real classrooms in real time.

The following key points, adapted from GAIMME (2016), should be considered when planning to engage students in learning through mathematical modeling.

MODELING (LIKE REAL LIFE) IS OPEN-ENDED AND MESSY.
It may seem like a good idea to help students by distilling a problem so they can immediately see the important factors to be considered. However, doing so prevents them from doing this on their own and robs them of the feelings of investment and accomplishment in their work. Also, mathematical models are not perfect and multiple models can provide very different results. Mathematician George Box summed this up beautifully when he said, "All models are wrong, but some are useful." (Box and Draper 1987, p. 424).

WHEN STUDENTS ARE MODELING, THEY MUST BE MAKING GENUINE CHOICES.
The best problems involve making decisions about things that matter to the students and help them see how using mathematics can help them make good, informed decisions.

START BIG, START SMALL, JUST START.
You may feel ready to jump in and make big changes, and if so, that is great! However, even small changes to things you already do in your classroom can encourage students to engage in mathematical modeling. To start small, choose a mathematical modeling task that you feel comfortable with – maybe one that you and your colleagues tackle while engaging in the mathematical modeling framework.

**ASSESSMENT SHOULD FOCUS ON THE PROCESS, NOT THE PRODUCT.**
Mathematical models (and the results they produce) are intimately tied to the assumptions made in creating the models. Assessment should be in service of helping students improve their ability to model, which will, in time, translate to a better product.

MODELING DOES NOT HAPPEN IN ISOLATION.
Whether students are working in teams, sharing ideas with the whole class, or going online to do research or collect data, modeling is not about working in a vacuum. The problems are challenging, and it helps to know you have support as you seek answers.

Mathematical Modeling Continuum

The Mathematical Modeling Continuum provides a visual and applicable flow map that illustrates how we can address 21st century (cognitive, intrapersonal, interpersonal) competencies domains through a progression of the four stages of mathematical problem development: Mathematics Problems, Word Problems, Application Problems and Mathematical Modeling Problems. The eventual objective of the continuum is to develop a process in which students learn to transfer knowledge in one situation and apply it to new situations.
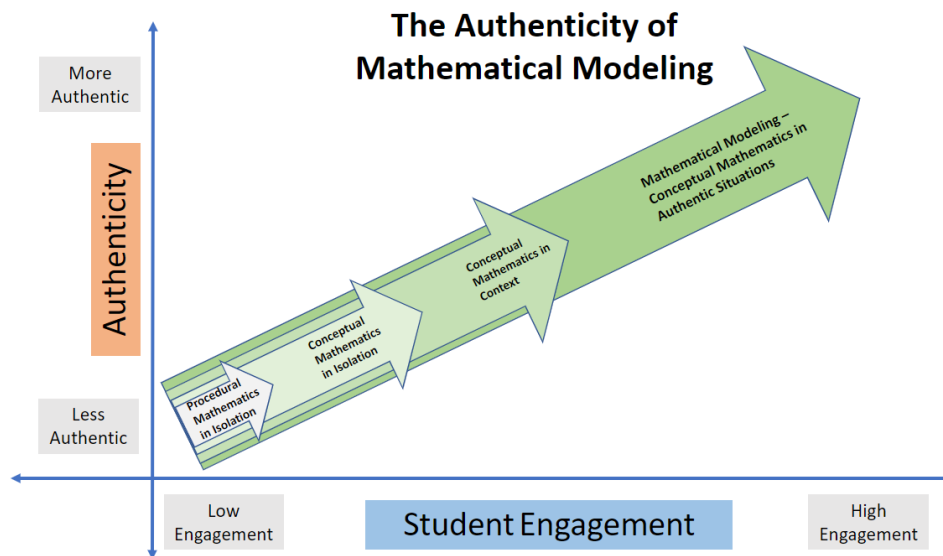
Supports for Learner Variability

Instructional Support Strategies

Within each learning plan in the instructional units, specific instructional support strategies are provided in the Student Learning Supports section. These strategies are largely organized into three categories: Supporting the Learning, Extending the Learning, and Language Supports. A description and rationale for these categories are written below; the actual strategies for support can be found within each specific learning plan.

Supporting the Learning
Teachers greatly influence how students perceive and approach the learning in the mathematics classroom. Even young students can learn to value struggle as an expected and natural part of learning, (Principles to Actions, 2014). To close gaps in mathematical understanding, a focus needs to be placed on the structure and teaching strategies implemented in classrooms. Within the learning plans in each unit, supports designated as Supporting the Learning within the Student Learning Support sections will include, but are not limited to: • intervention activities specific to the learning experiences within the learning plan • teacher actions from the High School Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan
Extending the Learning
According to Van De Walle, there are four basic strategies for adapting mathematics concepts for students who consistently demonstrate a solid understanding of the concepts of study. • *Acceleration* is characterized by self-paced learning and frequent exploration of similar topics but include higher-level thinking, more complex or abstract ideas and deeper levels of understanding or content. • *Enrichment* activities go beyond the topic of study to content that is not specifically a part of the grade level or course curriculum but is aligned with the lesson goals. • *Sophistication* provides a natural world view of mathematics when the level of complexity is increased or more depth is pursued. • *Novelty* introduces completely different materials from the regular curriculum. Within the learning plans in each unit, supports designated as Extending the Learning within the Student Learning Support sections will include, but are not limited to: • extension activities specific to the learning experiences within the task • instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GaDOE Talent Development Team

Language Supports
Teachers support students' language development in the context of mathematical sense-making through meaningful "reciprocal" interactions and discourse with others. As ELs explore and connect new math concepts, they will need many well-supported opportunities to use language in listening, speaking, reading, and writing (Baker et al., 2014). Within the learning plans in each unit, supports designated as Language Supports within the Student Learning Support sections will include, but are not limited to: - teacher actions from the English Language Proficiency for English (as a 2nd language) Learners section of the High School Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan   - Evidence-Based Instructional Strategies: collection of vetted GA Mathematics resources and evidence-based instructional strategies that highlight the benefits of hands-on, relevant experiences in Mathematics that support multilingual learners   - strategies and resources included in the document Mathematics Resources to Support English Learners found in the GaDOE mathematics resources provide specific evidence-based practices that indicate the benefits of hands-on, relevant learning experiences in the mathematics classroom.

Intervention Supports

INTERVENTION SUPPORTS FOR LEARNER VARIABILITY

Supports for learner variability can be found in each *Mathematics Learning Plan*. Supports will include, but are not limited to:

- intervention activities and English language proficiency resources specific to the learning experience are included within the learning plan

- teacher actions and resources from the High School Learner Variability Toolkit will assist educators as they support student success in mathematics

- extension activities specific to the learning experience within the task

- instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GADOE Gifted Department

INTERVENTION TABLE OF TASKS/ACTIVITIES

The Intervention Table below provides links to intervention tasks/activities specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from Georgia Secondary Numeracy Project.

Standard	Learning Objectives	Name of Intervention Task/Activity	Skills Addressed
A.DSR.10	A.DSR.10.4	[Forensic Formulas](#) [Matching Graphs and Situations, Using Rates of Change](#)	Relate rate of change to the slope of a graph.

*Tasks are aligned to the Georgia's K12 Mathematics Standards within the Georgia Secondary Numeracy Project which encompasses the identified skill.

Helpful Resources and Links

HELPFUL RESOURCES AND LINKS

- [K-12 Mathematics Glossary](#)
- [Make Mathematics Count, GA!](#)
- [Georgia Home Classroom](#)
- [Georgia Numeracy Project](#)
- [K-12 Mathematical Modeling Framework](#)
- [K-12 Statistical Reasoning Framework](#)
- [K-12 Mathematical Practices](#)
- [Tap Into Teen Minds](#)

Additional relevant links to resources can be found in each Mathematics Learning Plan.