Algebra:
Concepts and Connections

Unit 8:

Algebraic Connections to Geometric Concepts

Students will solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.

MATHEMATICS

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Unit Overview

Algebraic Connections to Geometric Concepts

UNIT OVERVIEW

Students will solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena. Students have prior experiences with the concepts of approximating radicals, calculating slopes, and graphing lines. Students will now use Algebra to model Geometric ideas by solving problems involving distance, midpoint, slope, area, and perimeter. Characteristics of these terms will be used to classify quadrilaterals in the coordinate plane. Students will use properties of lines cut by a transversal along with the distance formula to explore characteristics of quadrilaterals. Students will use coordinates to prove simple geometric theorems using the properties of distance, slope, and midpoints. Students will use distance formula and problem solve to calculate area and perimeter of special parallelograms and triangles.

Standards Addressed in this Unit

A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

- **A.MM.1.1:** Explain applicable, mathematical problems using a mathematical model.
- **A.MM.1.3:** Use units of measure (linear, area, capacity, rates, and time) as a way to make sense of conceptual problems; identify, use, and record appropriate units of measure within context, within data displays, and on graphs; convert units and rates using proportional reasoning given a conversion factor; use units within multi-step problems and formulas; interpret units of input and resulting units of output.
  A.MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.
- **A.MM.1.5:** Define appropriate quantities for the purpose of descriptive modeling.

A.GSR.3: Solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.

- **A.GSR.3.1:** Solve real-life problems involving slope, parallel lines, perpendicular lines, area, and perimeter.

- **A.GSR.3.2:** Apply the distance formula, midpoint formula, and slope of line segments to solve real-world problems.

Mathematical Practices

These 8 Mathematical Practices and the overarching Practice Standard are essential to the instruction in this unit.

A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.

A.MP.1: Make sense of problems and persevere in solving them.

A.MP.2: Reason abstractly and quantitatively.

A.MP.3: Construct viable arguments and critique the reasoning of others.

A.MP.4: Model with mathematics.

A.MP.5: Use appropriate tools strategically.

A.MP.6: Attend to precision.

A.MP.7: Look for and make use of structure.

A.MP.8: Look for and express regularity in repeated reasoning.

Algebra: Concepts and Connections Comprehensive Course Overview

The Comprehensive Course Overview for Algebra: Concepts and Connections provides additional details and information related to teaching and learning the mathematics standards in the course. To get more information, click on the link to the Algebra Course Overview below.

LINK TO ALGEBRA: CONCEPTS & CONNECTIONS COMPREHENSIVE COURSE LEVEL OVERVIEW

Professional Learning Videos

A series of professional learning videos have been developed by the Georgia Department of Education in partnership with Georgia Public Broadcasting to provide instructional support for classroom teachers and teacher leaders. These videos provide a detailed explanation of the standards and instructional suggestions for the classroom to help all learners master the content at high levels. The professional learning video(s) for the standards address in this unit can be found within the GPB Home Classroom and on the GaDOE webpage (www.gadoe.org/mathematics) and within www.georgiastandards.org.

Progressions

This unit gives students the opportunity to connect the Algebraic concepts that they have learned up to this point and connect them to geometric problems. This will prepare them for real-world concepts that they will see in Geometry. Students will start off with using the slope-intercept equation and the slope formula to solve problems involving parallel and perpendicular lines. Students will use what they know about the Pythagorean Theorem to find side lengths in the coordinate plane which in turn will lead to them learning the distance formula. They will further their use of the distance formula by finding the area and perimeter of shapes within the coordinate plane. Lastly, the students will use all of these concepts (slope, distance and midpoint) to identify shapes in the coordinate plane.

GA K-12 Learning Progressions - Use this document to provide a visual representation of the standards across all grade levels to make key connections among key concepts as students move from grade level to grade level (vertical alignment).

Grade 8 – Advanced Algebra: Concepts & Connections MATHEMATICS: LEARNING PROGRESSIONS				
Key Concepts	**Grade 8**	**Algebra: Concepts & Connections**	**Geometry: Concepts & Connections**	**Advanced Algebra: Concepts & Connections**
Shapes and Properties	• Introduction to Pythagorean Theorem and the converse	• Apply the distance formula, midpoint formula, and slope of line segments to solve real-world problems. • Apply slope criteria for parallel and perpendicular lines	• Develop and use precise definitions to prove theorems and solve geometric problems • Prove slope criteria for parallel and perpendicular lines	• Exploring trigonometry and the Unit Circle
Geometric Measurement	• Pythagorean Theorem to determine distance between two points • Volume of cones, cylinders, and spheres	• Use distance formula, midpoint formula, and slope to calculate perimeter and area of triangles and quadrilaterals.	• Volumes of prisms, cones, cylinders, pyramids, and spheres • Approximate volumes of irregular objects	• Solving geometric measurement problems using trigonometry

Unit Description

Algebraic Connections to Geometric Concepts

UNIT DESCRIPTION

In this unit, students will solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena. Students will learn how to do the following:

- Derive the distance formula through the use of Pythagorean theorem.
- Use coordinates, slope relationships, and distance formula to prove simple geometric theorems algebraically.
- Compute the perimeters of polygons using the coordinates of the vertices and the distance formula.
- Find the areas of rectangles and triangles using the coordinates of the vertices and the distance formula.
- Show that the slopes of parallel lines are the same.
- Show that the slopes of perpendicular lines are opposite reciprocals.
- Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.

In this unit, students will learn the relationships between slopes of parallel lines and between perpendicular lines and then use those relationships to write the equations of lines. They will also extend the use of the Pythagorean Theorem to the coordinate plane to introduce students to the distance formula, revisit definitions of polygons while using slope and distance on the coordinate plane and use knowledge of algebra and geometry concepts to find the area and perimeter of defined figures. In the end, students will solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.

This unit contains information and activities to teach the standards listed above using a conceptual, student-centered approach. The information presented here is a support for teachers in the implementation of the Effective Mathematics Teaching Practices and in engaging students in the Mathematical Practices.

The tasks and activities provide further guidance and support and are designed with several key features adding clarity for the implementation of these tasks and activities.

Evidence-Based Practices

EVIDENCE-BASED PRACTICES

What evidenced-based instructional practices could be employed to support the learner's growth and development in this unit?

For a more detailed description for each practice, please read the Algebra Guide for Effective Mathematics Instruction.

Evidence-Based Practice	Description	Additional Information (Graphic, Chart, Link)
21st Century Learning	The 21st century technology tools can be used to foster reasoning and sense-making with mathematical tasks.	
3-Act Math Tasks	A 3-Act Task is a mathematics task consisting of three distinct parts: an engaging and perplexing Act One, an information- and solution-seeking Act Two, and a solution discussion and a solution-revealing Act Three.	Effective Instructional Practices Guide
Collaborative Group work	Collaborative group work is encouraged in classrooms to help students make sense of mathematics and engage in productive classroom discourse in groups with their peers.	
Gamification	This instructional strategy finds a balance between games, which are considered 'fun' and voluntary, and learning. Gamification research reveals, however, that the components of well-designed games (goals, rules, feedback, autonomy, and voluntary participation) parallel those of well-designed learning experiences.	Learning and Games Unpacking the potential of educational gaming: A new tool for gaming research Measuring Problem Solving Skills via Stealth Assessment in an Engaging Video Game Scaffolding Game-Based Learning: Impact on Learning Achievements, Perceived Learning, and Game Experiences
Incorporating the 8 Mathematical Practices	The 8 Mathematical Practices are the critical overarching behaviors for each grade level. These eight practices should be incorporated in all lessons and activities for students.	

Modeling with Mathematics	Mathematical modeling is defined as the process of designing and revising representations to solve problems. This process is foundational for students to gain a deep understanding of mathematical concepts in context of real-life phenomena. For students to effectively model with mathematics, the problems must be presented in context. Students make sense of the problem, make decisions about what information is most important, create models using that information, perform operations using those models, interpret the results and validate conclusions. Mathematics is the language of the universe. Mathematics is everywhere. Therefore, this evidence-based practice requires for students to be equipped with the skills and knowledge necessary to use mathematics as a model to explain life around them, which is one of the ultimate purposes of mathematics.	Teaching Channel – Model with Mathematics Mathematical Modeling Missing the Promise of Mathematical Modeling (NCTM) Mathematical Modeling Framework Mathematical Modeling Continuum What is Mathematical Modeling?
Multiple Representations	Mathematical representations are visible or tangible productions – such as diagrams, number lines, graphs, arrangements of concrete objects or manipulatives, physical models, mathematical expressions, formulas and equations, or graphical depictions of mathematical ideas or relationships.	
Number Talks	A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation.	Number Talks Video Effective Instructional Practices Guide
Numeracy Intervention Resources	The use of formative assessments and intervention strategies will help students develop a strong knowledge of foundational skills and concepts in numeracy. The Georgia Numeracy Project offers specific intervention resources to support student learning and development of a solid foundation in numeracy.	Georgia Early Numeracy Project Overview Georgia Secondary Numeracy Project Overview

Patient Problem Solving	Patient problem solving deepens a learner's mathematical literacy. Mathematical literacy means having the ability to problem-solve, reason and analyze information. It is the ability to understand the "language" of mathematics, use numbers to help solve real-world problems, recognize patterns, and generalize pattern recognition to various contexts.	
Pattern Talks	Mathematics is everywhere in the world; therefore, classroom lessons and activities should help students see mathematics in the world and think about the patterns they see. It strengthens their learning and understanding.	Visual Patterns YouCubed Patterns Video
Positive Mathematical Mindsets and Productive Efforts of Learning	Promoting positive mathematical mindsets and productive efforts of learning gives students the time and space needed to work through mathematics problems.	Positive Mathematical Mindsets
Problem-Based Learning	Problem-based learning provides a structure for discovery that helps students internalize learning and leads to greater comprehension. Problem-based learning is student centered and develops both problem-solving strategies and content knowledge aimed at using the power of authentic problem solving to engage students.	"Problem-Based Learning" "New, Strong Evidence for Problem-Based Learning" Video-" Solving Real-World Issues Through Problem-Based Learning"
Project-Based Learning	Project-based learning is an authentic instructional model or strategy in which students plan, implement, and evaluate projects that are real-world applications beyond the classroom.	"What is Project-based learning?": PBL Works "Project-Based Learning Engages K-12 Students with Real-World Challenges" "Project-Based Learning Research Review: Evidence-Based Components of Success" (Edutopia) 21st Century Project Based Learning Video-" What Makes Project-Based Learning a Success?" Video-"Project-Based Learning: Explained"

Standards-Based Grading and Intervention Models	Interventions based on evidence-based practices and high leverage practices that include standards-based grading and intervention models should be incorporated in all classrooms K-12 to support increased student learning of the standards for each grade level.	
Visual Mathematics	Research conducted by Dr. Jo Boaler indicates that the human brains think about mathematics visually. Students who see mathematics visually understand the concepts more deeply.	Brains Think Math Visually Video
Which One Doesn't Belong?	In the Which One Doesn't Belong (WODB) activities, students are presented with four figures and asked to identify which one of the four figures does not belong with the other three figures. The focus is not on finding a singular correct answer, but rather, justifying the selection of their choice.	Which One Doesn't Belong?

Strategies for Teaching and Learning

STRATEGIES FOR TEACHING AND LEARNING

Some strategies for teaching and learning to help students master the concepts presented in this unit are identified below.

Instructional Strategies	Strategy for Implementation
Higher Order Questioning	Higher order questioning occurs throughout the entire lesson. Teachers should engage students through questioning to assess their level of understanding. This will act as a part of the formative assessment throughout the lesson allowing the teacher to make adjustments to the lesson as they progress.
Engaging/ Activating Strategy	Gets students actively thinking or making a connection with the material being presented that day. The activating strategy is apparent through the engage activity and may also occur in the explore activity.
Academic Discussions	Students should engage in academic discussions throughout the lesson. They should be the driver of the discussion based on the higher order questioning that occurs. Students should be questioning each other and justifying their solutions.
3-Read Protocol	The 3-Read protocol should be used to ensure students are comprehending the situations presented in the learning activities that are task driven. 1) The first read provides the ability for general understanding. Generally, what is the situation about? 2) The second read provides the ability to determine the quantities. What are the needed quantities that are provided? 3) The third read provides the what. What mathematical questions can we ask or what needs to be answered?
Differentiation	Activities are included according to students' unique learning needs providing students with higher academic capabilities the ability to be stretched and those who are struggling to get the appropriate support they may need to access the material.
Summary/Reflection	Teachers should summarize each of the lesson activities prior to dismissal which will allow them the ability to assess where their students are in the learning and provide students a time to reflect. A reflection activity is included for student use.

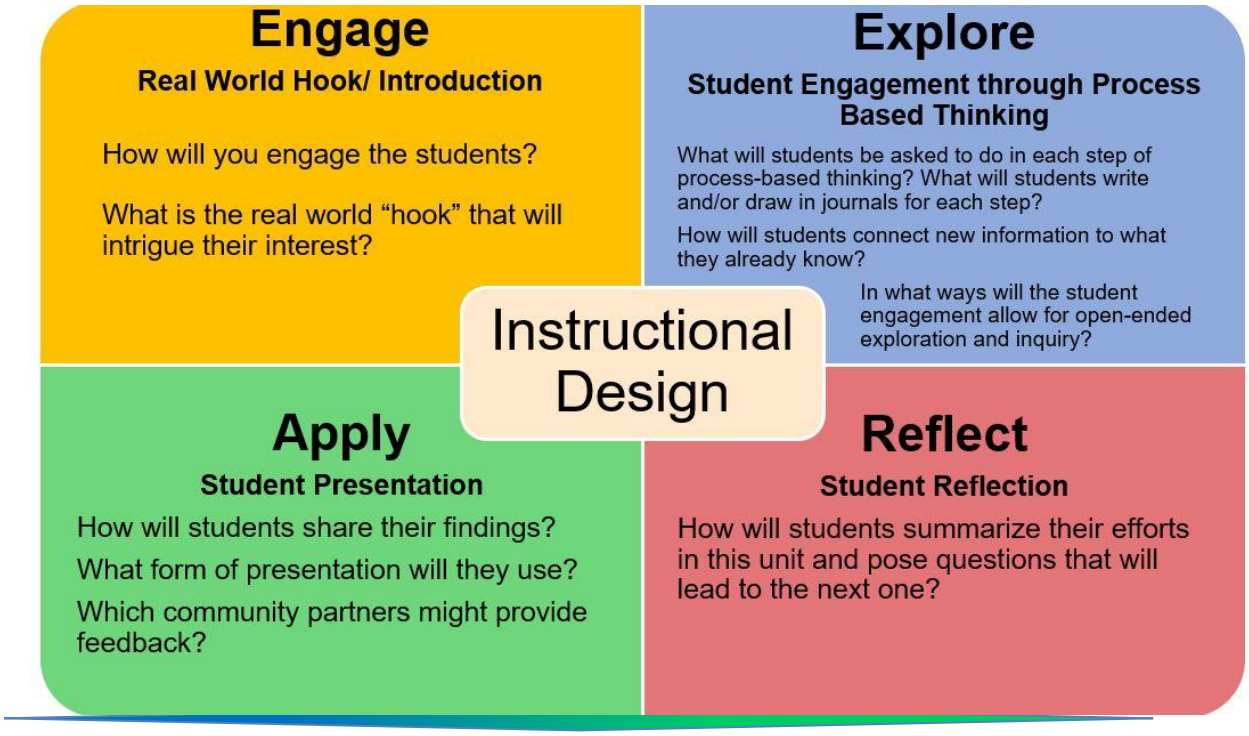
Graphic Organizer	The use of a graphic organizer allows students to organize their thinking. It allows them to visually categorize new information or review old information.

Instructional Design

INSTRUCTIONAL DESIGN DESCRIPTIONS

Instructional lessons included in this course follow the instructional design involving four sections: Engage, Explore, Apply, and Reflect. Specific descriptions for each section are included below.

- **Engage -** Within this section, the learning experiences include evidence-based instructional strategies that can be used as an introduction that mentally engages students to capture their interest, provides an opportunity to communicate what they know, and allows them to connect what they know to new ideas.
- **Explore -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to engage in hands-on activities to explore the new concept/big idea at a deep level.
- **Apply -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to apply what they have learned in a new situation to develop a deeper understanding of the big idea.
- **Reflect -** Within this section, the learning experiences include evidence-based instructional strategies that allow students the opportunity to review and reflect on their own learning and new understandings.



Relevant Academic Vocabulary

RELEVANT ACADEMIC VOCABULARY

An interactive K-12 Mathematics Glossary is now available as a reference tool for all mathematics teachers. Relevant academic vocabulary terms important to the content of this unit are included in the table below.

NOTE: This list is not intended as a vocabulary list for students, but as a reference for teachers that may be used to ensure precise language is used and encouraged by all.

area	perimeter
blueprint	perpendicular
coordinates	phenomena
distance	proof
distance formula	reciprocal
intersection	slope
line segment	slope relationships
midpoint	theorem
parallel	vertices

Concepts and Skills to Maintain

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Approximating radicals
- Calculating slopes of lines
- Graphing lines
- Writing equations for lines
- Number sense
- Computation with whole numbers and decimals, including application of order of operations
- Addition and subtraction of common fractions with like denominators
- Applications of the Pythagorean Theorem
- Graphing on a coordinate plane
- Operations with radicals

Instructional Learning Plans

What learning activities can be used to teach the standards associated with this unit?

Instructional Learning Plan*	Duration	Learning Tasks	Standards/Learning Expectations
Home Design Teacher Guidance Blackline Masters Student Reproducibles	**1 – 2 days**	• Sample Floor Plans & Smart Draw Technology • Fixer Upper	**A.GSR.3** • A.GSR.3.1 • A.GSR.3.2 **A.MM.1** • A.MM.1.1 • A.MM.1.4 • A.MM.1.5 **A.MP**
Learning Plan Description: In this learning plan, students will embark on a journey of real-life problem solving in the context of building their dream home. They will apply their understanding of Pythagorean theorem to discover the distance formula and calculate area and perimeter of shapes solving practical problems encountered in home design. By calculating the area and perimeter of shapes, students will gain the skills needed to create efficient and functional living spaces.			
Design Challenges Teacher Guidance Student Reproducibles	**1 – 2 days**	• Which One Doesn't Belong • Polygon Card Sort • Classifying Triangles • Classifying Polygons	**A.GSR.3** • A.GSR.3.1 • A.GSR.3.2 **A.MM.1** • A.MM.1.1 • A.MM.1.4 **A.MP**
Learning Plan Description: In this learning plan, students will face a set of challenges where they must create shapes (triangles and quadrilaterals) in order to prepare themselves for making sketches of their dream home at the end of the unit. Students will focus on finding the area of and perimeter quadrilaterals and triangles using the distance formula on a coordinate grid. As students begin to get comfortable with using the distance formula, it will help them later with their dream home.			
City Design Teacher Guidance Student Reproducibles	**1 – 2 days**	• Which One Doesn't Belong? • Desmos: Slopes of Parallel and Perpendicular Lines • Designing a City	**A.GSR.3** • A.GSR.3.1 • A.GSR.3.2 **A.MM.1** • A.MM.1.1 • A.MM.1.4 **A.MP**
Learning Plan Description: In this learning plan, students will engage in a guided discovery activity to apply the relationship between the slopes of parallel lines and the slopes of perpendicular lines. Through this task, students will verify geometric relationships in the coordinate plane using algebraic thinking. They will focus on applying slopes of parallel and perpendicular lines in creating a design for a city. Students will deepen their understanding of the connections between slopes, parallel lines, and perpendicular lines in the coordinate plane while building on their skills for determining area and perimeter of shapes on a coordinate grid.			

Enhancing the City Design Teacher Guidance Student Reproducibles	**2 – 3 days**	• Traffic Engineering and Design • What's the Midpoint?! • City Design Details – Part 1 • City Design Details – Part 2	**A.GSR.3** • A.GSR.3.2 **A.MM.1** • A.MM.1.1 • A.MM.1.4 • A.MM.1.5 **A.MP**
Learning Plan Description: In this learning plan, students will explore midpoint through architecture and design. Students will explore the midpoint formula to make sense of the concept in order to solve a real-life problem. Students can apply their understanding of the distance and midpoint formulas to solve real-world problems. To advance their knowledge, this learning plan will extend their understanding through real-life application. Using a figure in the coordinate plane, students will be able to find a location using distance or midpoint.			

Numeracy Development Intervention Supports

Georgia Numeracy Project Resources

Georgia Secondary Numeracy Project

The Georgia Secondary Numeracy Project is a numeracy development resource provided by the Georgia Department of Education, which introduces teachers and teacher leaders to the trajectory by which learners acquire and build a solid foundation in numeracy which ultimately supports algebra readiness.

The Georgia Secondary Numeracy Project consists of a two-part universal screener:

- Part 1 of the universal screener is the Diagnostic Assessment. This assessment assesses students' strategy proficiency across 3 domains, Addition and subtraction, multiplication and division, and proportions and ratios. Building upon the strategy stages from the Georgia Early Numeracy Project, the Georgia Secondary Numeracy Project follows this progression.
- The second component of the universal screener is the Written Assessment. Using the Overall Strategy Stage from the Diagnostic Interview, students begin the Written Assessment on the Part equivalent to the Overall Strategy Stage from the Diagnostic Interview. The domains assessed on the Written Assessment are: Relational & Functional Reasoning, Patterning & Algebraic Reasoning, Statistical & Probability Reasoning and Geometric, Spatial & Measurement Reasoning.

In alignment with Georgia's Tiered Supports for Students, the Georgia Secondary Numeracy Project provides tools to target specific skills and provide tiered supports and interventions. To address the identified skills, the Georgia Secondary Numeracy provides numeracy development intervention tasks and activities. Within each unit in the grade level/course, intervention tables are provided to identify specific tasks and activities aligned to the standards and learning objectives discussed within the unit.

The Georgia Secondary Numeracy Project is the perfect complement to Georgia's K-12 Mathematics Standards, aligning with the multiple big ideas and aiding in the development of mathematical reasoning. The Georgia Secondary Numeracy Project supports the various mathematical concepts in Grade 8 through Advanced Algebra: Concepts & Connections.

College and Career Readiness

The GaDOE Roadmap for Reimaging K-12 Education provides teachers and leaders the opportunity to reimagine the learning experiences for students to ensure they are prepared for the future. The goal is to match their passions, their interests, and their abilities with the opportunities they are provided in school and beyond. All students have a uniqueness that can be nurtured and fostered, which can grow into something that is amazing. The goal of the Georgia Department of Education is to ensure that all students have any door of opportunity they desire open for them. Students begin exploring careers and preparing for post-secondary opportunities as early as Kindergarten. As students matriculate through elementary school and middle school, their exploratory opportunities for careers become more refined and focused to assist them with decision-making related to course taking options at the secondary level. Helping students refine their college and career interests, coupled with their strengths and goals will assist them with a great start in identifying post-secondary opportunities. Also, specific course pathways can be selected to ensure they build the foundational knowledge necessary for their college and career pathway of interest.

Providing opportunities for internships and collaboration with local business and community partners will also help students make connections between what they are learning in mathematics class with careers that use the mathematics can be very powerful and eye-opening for the learners. These opportunities should be provided to help students on their quest to become prepared for their future, they are being prepared for life.

At the secondary level, students have the flexibility to participate in personalized mathematics pathways that allow them to deeply explore advanced mathematics content in data science, statistics, quantitative reasoning, and/or calculus based on their future goals and interests. As students participate in advanced-level coursework in high school through the personalized pathways, opportunities should be provided for them to see the connections and experience the mathematics content in the real-world.

Interdisciplinary Teaching and Learning Support

Process-Based Thinking

Process thinking is an important element of interdisciplinary teaching and learning. Process thinking has six components:

1. **Intellectual Challenge and Accomplishment** – this component focuses on developing students' capacity to learn deeply, think critically, and strive for excellence.
2. **Authenticity** – this component focuses on students working on projects that are meaningful and relevant to their community, their lives, and their future.
3. **Public Product** – students' work is publicly displayed, discussed, and critiqued.
4. **Collaboration** – this component focuses on building students' capacity to collaborate with other students in person or online and/or receive guidance from adult mentors and experts.
5. **Project Management** – students use a project management process that enables them to proceed effectively from project initiation to completion.
6. **Reflection** – students reflect on their work and their learning throughout the project.

The six components of process-based thinking connect nicely to the 8 Mathematical Practices that are embedded in all instructional units. These are a part of the Essential Instructional Guidance included for all grade levels and high school courses. Grade 8 lessons include opportunities for students to engage in process-based thinking through exploration and sense-making of the mathematics content standards.

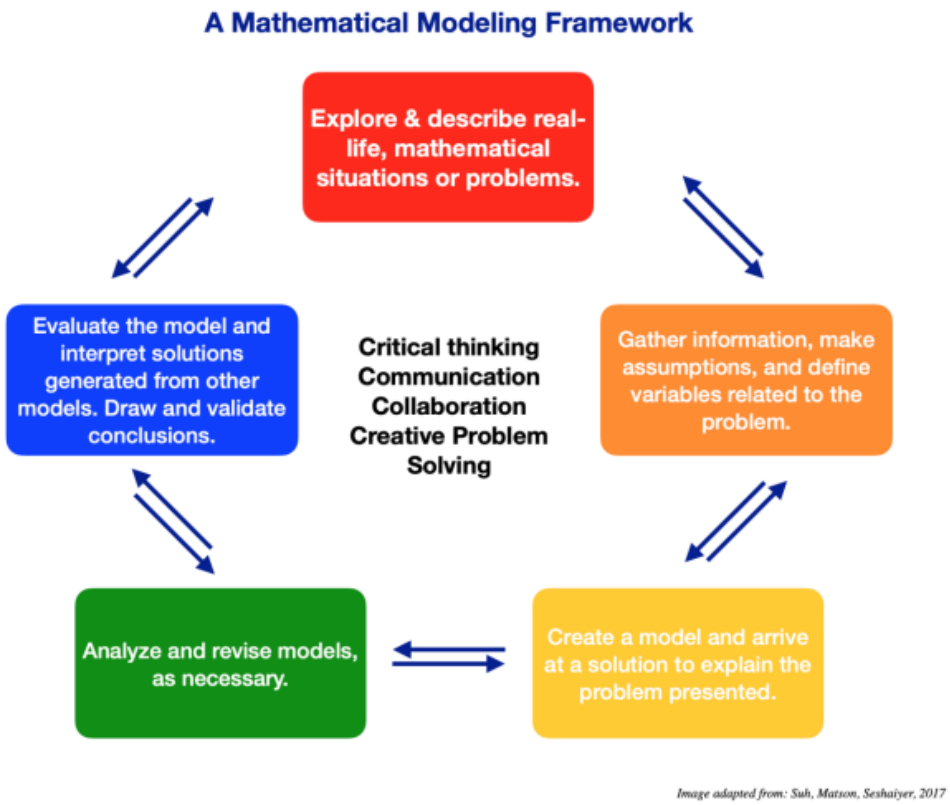
Mathematical Modeling Overview

Mathematical modeling can be explored through interdisciplinary teaching and learning. One instructional model for interdisciplinary teaching and learning is project-based learning. There are six powerful project-based instructional practices:

1. Plan Authentic, Intellectually Demanding Project-Based Learning Units Where Students Master Significant Content and Skills
2. Utilize Sustained, In-Depth Inquiry
3. Engage Students in a Collaborative Problem-Solving/Design Process
4. Foster a Classroom Environment That Supports Student Ownership of Learning
5. Engage in Ongoing and Purposeful Feedback, Revision, and Reflection
6. Include Community Partners in Project Planning, Implementation, and Reflection

Mathematical Modeling Framework

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create. The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.



The Mathematical Modeling Framework offers some insight into what modeling with mathematics looks like when implemented in K-12 classrooms.

- Real-life, mathematical situations or problems are investigated.
- Students gather information, make assumptions, and define unknowns (variables).
- Mathematical models are created and used to arrive at a solution to explain the real-life, mathematical situation or problem.
- Models are analyzed and revised as needed.
- Models are evaluated by students. Solutions using different models are interpreted and conclusions are drawn and validated.

When choosing mathematical modeling tasks and activities, keep in mind that these tasks should (1) be interesting and/or important for students to experience, (2) exemplify specific components of the modeling cycle, (3) be doable by real students in real classrooms in real time.

The following key points, adapted from GAIMME (2016), should be considered when planning to engage students in learning through mathematical modeling.

MODELING (LIKE REAL LIFE) IS OPEN-ENDED AND MESSY.
It may seem like a good idea to help students by distilling a problem so they can immediately see the important factors to be considered. However, doing so prevents them from doing this on their own and robs them of the feelings of investment and accomplishment in their work. Also, mathematical models are not perfect and multiple models can provide very different results. Mathematician George Box summed this up beautifully when he said, “All models are wrong, but some are useful.” (Box and Draper 1987, p. 424).

WHEN STUDENTS ARE MODELING, THEY MUST BE MAKING GENUINE CHOICES.
The best problems involve making decisions about things that matter to the students and help them see how using mathematics can help them make good, informed decisions.

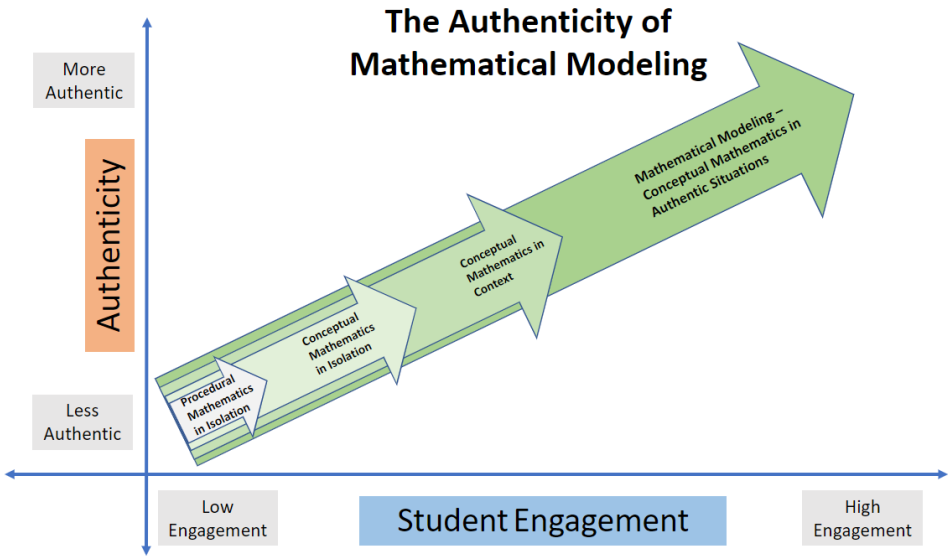
START BIG, START SMALL, JUST START.
You may feel ready to jump in and make big changes, and if so, that is great! However, even small changes to things you already do in your classroom can encourage students to engage in mathematical modeling. To start small, choose a mathematical modeling task that you feel comfortable with – maybe one that you and your colleagues tackle while engaging in the mathematical modeling framework.

ASSESSMENT SHOULD FOCUS ON THE PROCESS, NOT THE PRODUCT.
Mathematical models (and the results they produce) are intimately tied to the assumptions made in creating the models. Assessment should be in service of helping students improve their ability to model, which will, in time, translate to a better product.

MODELING DOES NOT HAPPEN IN ISOLATION.
Whether students are working in teams, sharing ideas with the whole class, or going online to do research or collect data, modeling is not about working in a vacuum. The problems are challenging, and it helps to know you have support as you seek answers.

Mathematical Modeling Continuum

The Mathematical Modeling Continuum provides a visual and applicable flow map that illustrates how we can address 21st century (cognitive, intrapersonal, interpersonal) competencies domains through a progression of the four stages of mathematical problem development: Mathematics Problems, Word Problems, Application Problems and Mathematical Modeling Problems. The eventual objective of the continuum is to develop a process in which students learn to transfer knowledge in one situation and apply it to new situations.



Supports for Learner Variability

Instructional Support Strategies

Within each learning plan in the instructional units, specific instructional support strategies are provided in the Student Learning Supports section. These strategies are largely organized into three categories: Supporting the Learning, Extending the Learning, and Language Supports. A description and rationale for these categories are written below; the actual strategies for support can be found within each specific learning plan.

Supporting the Learning
Teachers greatly influence how students perceive and approach the learning in the mathematics classroom. Even young students can learn to value struggle as an expected and natural part of learning, (Principles to Actions, 2014). To close gaps in mathematical understanding, a focus needs to be placed on the structure and teaching strategies implemented in classrooms. Within the learning plans in each unit, supports designated as Supporting the Learning within the Student Learning Support sections will include, but are not limited to: • intervention activities specific to the learning experiences within the learning plan • teacher actions from the High School Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan
Extending the Learning
According to Van De Walle, there are four basic strategies for adapting mathematics concepts for students who consistently demonstrate a solid understanding of the concepts of study. • *Acceleration* is characterized by self-paced learning and frequent exploration of similar topics but include higher-level thinking, more complex or abstract ideas and deeper levels of understanding or content. • *Enrichment* activities go beyond the topic of study to content that is not specifically a part of the grade level or course curriculum but is aligned with the lesson goals. • *Sophistication* provides a natural world view of mathematics when the level of complexity is increased or more depth is pursued. • *Novelty* introduces completely different materials from the regular curriculum. Within the learning plans in each unit, supports designated as Extending the Learning within the Student Learning Support sections will include, but are not limited to: • extension activities specific to the learning experiences within the task • instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GaDOE Talent Development Team

Language Supports
Teachers support students' language development in the context of mathematical sense-making through meaningful "reciprocal" interactions and discourse with others. As ELs explore and connect new math concepts, they will need many well-supported opportunities to use language in listening, speaking, reading, and writing (Baker et al., 2014). Within the learning plans in each unit, supports designated as Language Supports within the Student Learning Support sections will include, but are not limited to: • teacher actions from the English Language Proficiency for English (as a 2nd language) Learners section of the High School Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan • Evidence-Based Instructional Strategies: collection of vetted GA Mathematics resources and evidence-based instructional strategies that highlight the benefits of hands-on, relevant experiences in Mathematics that support multilingual learners • strategies and resources included in the document Mathematics Resources to Support English Learners found in the GaDOE mathematics resources provide specific evidence-based practices that indicate the benefits of hands-on, relevant learning experiences in the mathematics classroom.

Intervention Supports

INTERVENTION SUPPORTS FOR LEARNER VARIABILITY

Supports for learner variability can be found in each *Mathematics Learning Plan*. Supports will include, but are not limited to:

- intervention activities and English language proficiency resources specific to the learning experience are included within the learning plan
- teacher actions and resources from the High School Learner Variability Toolkit will assist educators as they support student success in mathematics
- extension activities specific to the learning experience within the task
- instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GADOE Gifted Department

INTERVENTION TABLE OF TASKS/ACTIVITIES

The Intervention Table below provides links to intervention tasks/activities specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from Georgia Secondary Numeracy Project.

Standard	Learning Objectives	Name of Intervention Task/Activity	Skills Addressed
A.GSR.3	A.GSR.3.1	Polygraph: Figure it Out Matching Graphs and Situations, Using Rates of Change Forensic Formulas Proofs and Puzzles	Relate rate of change to the slope of a graph.
A.GSR.3	A.GSR.3.2	Deriving the Distance Formula Distance Formula	Exploring the distance formula

*Tasks are aligned to the Georgia's K12 Mathematics Standards within the Georgia Secondary Numeracy Project which encompasses the identified skill.

Helpful Resources and Links

HELPFUL RESOURCES AND LINKS

- K-12 Mathematics Glossary
- Make Mathematics Count, GA!
- Georgia Home Classroom
- Georgia Numeracy Project
- K-12 Mathematical Modeling Framework
- K-12 Statistical Reasoning Framework
- K-12 Mathematical Practices
- Tap Into Teen Minds

Additional relevant links to resources can be found in each Mathematics Learning Plan.