GEORGIA'S K-12 MATHEMATICS STANDARDS

Mathematics Teaching and Learning Resources

ALGEBRA: CONCEPTS & CONNECTIONS

Comprehensive Course Overview

MATHEMATICS

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SECTION 1: Course Description

In Algebra: Concepts & Connections, instructional time should regularly incorporate the 8 Mathematical Practices, the Framework for Statistical Reasoning, and the Mathematical Modeling Framework through four big ideas of content: (1) numerical reasoning, (2) functional & graphical reasoning, (3) patterning and algebraic reasoning, and (4) geometric and spatial reasoning. This course is designed as the first course in a three-course series. Students will apply their algebraic and geometric reasoning skills to make sense of problems involving algebra, geometry, bivariate data, and statistics. This course focuses on algebraic, quantitative, geometric, graphical, and statistical reasoning. In this course, students will continue to enhance their algebraic reasoning skills when analyzing and applying a deep understanding of linear functions, sums and products of rational and irrational numbers, systems of linear inequalities, distance, midpoint, slope, area, perimeter, nonlinear equations and functions, quadratic expressions, equations and functions, exponential expressions, equations, and functions, and statistical reasoning. *The identified prerequisite for this course is Grade 8 Mathematics.*

SECTION 2: Curriculum Map & Pacing

Interactive Curriculum Map

Georgia's K-12 Mathematics Standards
ALGEBRA: CONCEPTS & CONNECTIONS

Semester 1				Semester 2				
Unit 1	**Unit 2**	**Unit 3**	**Unit 4**	**Unit 5**	**Unit 6**	**Unit 7**	**Unit 8**	**Unit 9**
Modeling Linear Functions	Analyzing Linear Inequalities	Investigating Rational and Irrational Numbers	Modeling and Analyzing Quadratic Functions	Modeling and Analyzing Exponential Expressions & Equations	Analyzing Exponential Functions	Investigating Data	Algebraic Connections to Geometric Concepts	Culminating Capstone Unit
Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	Interdisciplinary Connection	
3 – 4 weeks	**1 – 2 weeks**	**1 – 2 weeks**	**6 – 7 weeks**	**2 – 3 weeks**	**4 – 5 weeks**	**3 – 4 weeks**	**2 – 3 weeks**	**1 - 2 weeks**
A.FGR.2 A.MM.1 A.MP.1-8	A.PAR.4 A.MM.1 A.MP.1-8	A.NR.5 A.MM.1 A.MP.1-8	A.PAR.6 A.FGR.7 A.MM.1 A.MP.1-8	A.PAR.8 A.MM.1 A.MP.1-8	A.FGR.9 A.MM.1 A.MP.1-8	A.DSR.10 A.MM.1 A.MP.1-8	A.GSR.3 A.MM.1 A.MP.1-8	ALL STANDARDS A.MM.1 A.MP.1-8

Ongoing interdisciplinary learning to impact the community and to explain real-life phenomena

The concepts in each unit are presented based on a logical, mathematical progression. Each unique unit in sequence builds upon the previous unit.

The Framework for Statistical Reasoning, Mathematical Modeling Framework, and the K-12 Mathematical Practices should be taught throughout the units.

Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

Key for Course Standards: MP: Mathematical Practices, **MM:** Mathematical Modeling, **NR:** Numerical Reasoning, **FGR:** Functional & Graphical Reasoning, **GSR:** Geometric & Spatial Reasoning, **PAR:** Patterning & Algebraic Reasoning, **DSR:** Data & Statistical Reasoning

Click here to access the entire Algebra: Concepts & Connections Curriculum Map.

SECTION 3: Background Research & Support

Ensuring Access to a Well-Rounded Mathematics Education

Georgia's Plan for Ensuring Access to a Well-Rounded Mathematics Education was developed based on collaborative work among various district leadership teams and university partners. The theory of action leaders used to develop this action plan includes actions for district leaders, school leaders, teachers, and students in order to ensure that each student entrusted in our educational care is a prepared learner for life. The specific goals outlined in the plan are:

GOAL 1: Prepare, hire, develop and support high-quality mathematics teachers for all students.

GOAL 2: Create mathematics learning environments with established supports, structures and mindsets to honor each student's uniqueness, experiences, background knowledge, and interests in order to advance their learning.

GOAL 3: Provide a prevention and intervention infrastructure to ensure that each learner is provided with the necessary support to succeed.

GOAL 4: Recognize, understand, and value each learner's individual uniqueness.

For a detailed description of each goal and the desired student outcome of accomplishing each goal, please refer to **Georgia's Plan for Ensuring Access to a Well-Rounded Mathematics Education.**

Mathematical Mindsets

Growth mindset was pioneered by Carol Dweck, Lewis, and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability (from With Math I Can).

You can learn how to use the power of growth mindset for yourself and your students in the resources below:

- Fostering Positive Mathematical Mindsets
- Youcubed: Inspire ALL Students with Open, Creative, Mindset Mathematics

Mathematical Reasoning

Mathematical reasoning is:

- making sense of mathematical ideas.
- a critical skill that enables students to analyze and solve a given problem.
- the process of converting information into mathematical language and then apply logic to determine desired results.
- an argument made to justify one's process, procedure, or conjecture, to create strong conceptual foundations and connections, in order for students to be able to process new information.
- a way of thinking that moves students beyond memorizing facts, towards thinking beyond the rules and procedures to forming their own questions and conjectures.
- teaching that requires planning well-developed tasks and pre-planned questions, to allow students to communicate at higher levels of comprehension that mathematical reasoning requires.
- much more than asking, "Why?".

In classrooms that promote mathematical reasoning, students are engaged and building confidence as they collaborate and think creatively and critically to solve problems about the world around them. Students are presented with real-life phenomena, encouraged to ask mathematical questions to build logical sense of that phenomena, and engage with mathematical concepts while exploring those real-life phenomena within the key competencies/course standards for each grade level/course. Through this approach to teaching and learning, the mathematical concepts are addressed within a cluster of conceptual ideas rather than isolated skills.

Engaging students in the process of mathematical modeling and the Framework for Statistical Reasoning promotes mathematical reasoning. These two frameworks build on students' natural curiosity and offer them the opportunity to make sense of the world around them, make predictions, and/or make informed decisions. These skills prepare students for their chosen career and life in a fast-paced, ever-changing world.

Developing Mathematical Thinking

In mathematics teaching and learning, we encourage students to see grapple with unfamiliar content, critically think about mathematics in the world around them, think like mathematicians, look at numbers before they calculate, and think conceptually rather than perform rote procedures without understanding. Students can and do construct their own strategies for problem-solving, and when they are allowed to make sense of mathematics in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others." By changing the way we teach, we are asking teachers to think mathematically, too. In the process of changing the way mathematics is taught, five key strategies should be implemented to increase student understanding in mathematics classrooms: (1) Allow for students to grapple with the unfamiliar content and engage in academic discourse; this perseverance leads to a deeper understanding and confidence that comes leading to student agency. (2) Encourage questioning and make space for curiosity. (3) Emphasize conceptual understanding over procedure. (4) Provide authentic problems that increase students' drive to engage with math. (5) Always share positive attitudes and energy about learning mathematics.

Computational Strategies

In mathematics, the emphasis is on the reasoning and thinking about the quantities within mathematical contexts. Specific mathematics strategies for teaching and learning are not mandated by the Georgia Department of Education or assessed on state or federally mandated tests. Students may solve problems in different ways and have the flexibility to choose a mathematical strategy that allows them to make sense of and strategically solve problems using efficient methods that are most comfortable for and makes sense to them. It is critical that teachers and parents remain partners to help each child grow to become a mathematically literate citizen. These standards preserve and affirm local control and flexibility. Area models, tape diagrams (bar models), number line representations, graphical displays, equations, tables, expressions, equations, and inequalities are a few examples of ways students communicate their strategic thinking in a written form.

Effective Mathematics Teaching Practices

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically. The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using instructional practices that are effective in developing mathematics learning for all students. The eight Mathematics Teaching Practices below describe the essential teaching skills derived from the research-based learning principles, as well as other knowledge of mathematics teaching that has emerged over the last two decades (NCTM, 2014).

1. Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

7. Support efforts of learning in mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in meaningful learning tasks as they grapple with mathematical ideas and relationships.
8. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

A deeper explanation of each of the 8 Effective Mathematics Teaching Practices can be found below and in *Principles to Actions* from NCTM.

Effective Mathematics Teaching Practices Teacher and Student Behaviors

Mathematics Teaching Practices	What is the teacher doing?	What are students doing?
1. Establish mathematics goals to focus learning.	• Consider broader goals, as well as the goals of the actual lesson, including the following: • What is to be learned? • Why is the goal important? • Where are students coming from? Where do students need to go? • How can learning be extended?	• Make sense of new concepts and skills; including connections to concepts/big ideas learned in previous grades. • Experience connections among the strands, overall and specific expectations. • Deepen their understanding and expect mathematics to make sense.
2. Implement tasks that promote reasoning and problem solving.	• Chooses task that: ○ are built on current student understandings. ○ have various entry points with multiple ways for the problems to be solved. ○ are interesting to students (e.g., evolve from students' thinking; connect to real world mathematics)	• Work to make sense out of the task and persevere in solving problems. • Use a variety of models and materials to make sense of the mathematics in the task. • Convince themselves and others the answer is reasonable.
3. Use and connect mathematical representations.	• Uses tasks that allow students to use a variety of representations. • Encourages the use of different representations, including concrete models, pictures, words, and numbers, that support students in explaining their thinking and reasoning.	• Use materials to make sense out of problem situations. • Connect representations to mathematical ideas and structures of big ideas, including operational sense with whole numbers, fractions, and decimals.

4. Facilitate meaningful mathematical discourse.	• Engages students in explaining their mathematical reasoning in small groups and classroom situations. • Facilitates discussion among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Explain the ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of others. • Ask questions of others to make sense of their ideas.
5. Pose purposeful questions.	• Asks questions that build on and extend student thinking. • Facilitates discussion among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Think more deeply about the process of the mathematics rather than simply focusing on the answer. • Listen to and comment on the explanations of others in the class.
6. Build procedural fluency from conceptual understanding.	• Provides opportunities for students to reason about mathematical ideas. • Expects students to explain why their strategies work. • Connects student methods to efficient procedures as appropriate.	• Understand and explain the procedures they are using and why they work. • Use a variety of strategies to solve problems and make sense of the mathematical tasks. • Do not rely on shortcuts or tricks to do mathematics.
7. Support efforts of learning in mathematics.	• Supports students in building their understanding without showing and telling a procedure but rather focusing on the important mathematical ideas. • Asks questions that scaffold and advance student thinking. • Builds questions and plans lessons based on important student misconceptions rather than focusing on the correct answer. • Recognize the importance of effort as students work to make sense of new ideas.	• Stick to tasks and recognize that effortful learning is part of making sense. • Ask questions that will help to better understand the task. • Support each other with ideas rather than telling others the answer or how to solve the problem.

8. Elicit and use evidence of student thinking.	• Determines what to look for in gathering evidence of student learning. • Poses questions and answers student questions that provide information about student understanding and reasoning. • Uses evidence to determine next steps of instruction.	• Accept reasoning and understanding are as important as the answer to the problem. • Use mistakes and misconceptions to rethink understanding. • Ask questions to clarify confusion or misunderstanding. • Assess progress toward developing mathematical understanding.

Adapted from John Hattie's (2017, p. 244) summation from Principles to Action (National Council of Teachers of Mathematics, 2014)

The *Algebra: Concepts & Connections Guide for Effective Mathematics Instruction* provides additional resources for evidence-based practices that support student learning.

SECTION 4: Essential Instructional Guidance

Understanding the Mathematical Practices

Mathematical Practices are listed with each grade or course mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The Mathematical Practices describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Mathematical Practices – *These mathematical practices describe how students should engage with the mathematics content for the course. Developing these habits of mind builds students' capacity to become mathematical thinkers.*		
A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.		
	Expectations	**Evidence of Student Learning**
A.MP.1	**Make sense of problems and persevere in solving them.**	*Relevance and Application* ● In Algebra: Concepts & Connections, students examine real world problems and look for entry points to its solution. ● Students analyze givens, constraints, relationships, and goals. ● Students make conjectures about the meaning of the solution and plan a solution pathway. ● Students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. ● Students monitor and evaluate their progress and change course if necessary. ● Students explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. ● Students understand the approaches of others to solving complex problems and identify correspondences between different approaches. *Fundamentals* ● Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?" "Does this make sense?", and "Can I solve the problem in a different way?"
A.MP.2	**Reason abstractly and quantitatively.**	*Relevance and Application* ● In Algebra: Concepts & Connections, students should seek to make sense of quantities and their relationships in problem situations. ● Students should be able to abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbol involved. ● Students should be able to use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

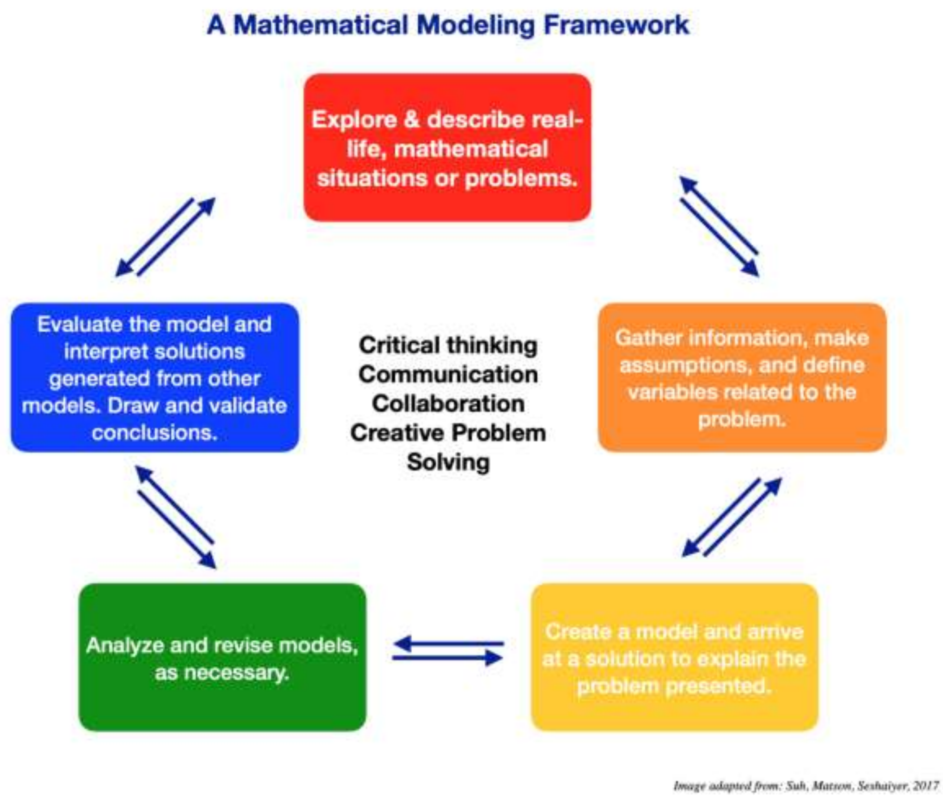
A.MP.3	**Construct viable arguments and critique the reasoning of others.**	*Relevance and Application* • In Algebra: Concepts & Connections, students use stated assumptions, definitions, and previously established results in constructing arguments. • Students make conjectures and build a logical progression of statements to explore the truth of their conjectures. • Students analyze situations by breaking them into cases and can recognize and use counterexamples. • Students justify their conclusions, communicate them to others, and respond to the arguments of others. • Students reason inductively about data, making plausible arguments that consider the context from which the data arose. • Students compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is. *Fundamentals* • Students should further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?” “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
A.MP.4	**Model with mathematics.**	*Relevance and Application* • In Algebra: Concepts & Connections, students solve problems arising in everyday life. • Students make assumptions/approximations to simplify a complicated situation. • Students identify important quantities and map their relationships using diagrams, graphs, flowcharts, and formulas. • Students analyze relationships mathematically to draw conclusions. • Students interpret their mathematical results in context and reflect on the reasonableness of their results, possibly improving the model if needed. *Fundamentals* • A student might use a function to describe how one quantity of interest depends on another.
A.MP.5	**Use appropriate tools strategically.**	*Relevance and Application* • Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful to explore and deepen their understanding of concepts. • Students identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. • Students use technological tools to explore and deepen their understanding of concepts. *Fundamentals* • Tools may include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, or computer software. • Students analyze graphs of functions and solutions generated using a graphing calculator.
A.MP.6	**Attend to precision.**	*Relevance and Application* • In Algebra: Concepts & Connections, students communicate precisely to others by using clear definitions in discussions with others and in their own reasoning. • Students state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. • Students calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. • Students examine claims and make explicit use of definitions.

A.MP.7	**Look for and make use of structure.**	*Relevance and Application* ● Students look closely to discern a pattern or structure. ● Students see complex structures, such as some algebraic expressions, as single objects or as being composed of several objects. *Examples* ● In the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$. ● Students recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. ● Students can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
A.MP.8	**Look for and express regularity in repeated reasoning.**	*Relevance and Application* ● In Algebra: Concepts & Connections, students notice if calculations are repeated and look both for general methods and for shortcuts. ● Students work to solve problems, derive formulas, or make generalizations, and maintain oversight of the process, while attending to the details. ● Students continually evaluate the reasonableness of their intermediate results.

SECTION 5: Mathematical Modeling

Mathematical Modeling Framework

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create. The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.)



The Mathematical Modeling Framework offers some insight into what modeling with mathematics looks like when implemented in K-12 classrooms.

- Real-life, mathematical situations or problems are investigated.
- Students gather information, make assumptions, and define unknowns (variables).
- Mathematical models are created and used to arrive at a solution to explain the real-life, mathematical situation or problem.
- Models are analyzed and revised as needed.

- Models are evaluated by students. Solutions using different models are interpreted and conclusions are drawn and validated.

When choosing mathematical modeling tasks and activities, keep in mind that these tasks should (1) be interesting and/or important for students to experience, (2) exemplify specific components of the modeling cycle, (3) be doable by real students in real classrooms in real time.

As you begin engaging your students in learning through mathematical modeling, please note the following:

MODELING (LIKE REAL LIFE) IS OPEN-ENDED AND MESSY.
It may seem like a good idea to help students by distilling a problem so they can immediately see which are the important factors to be considered. However, doing so prevents them from doing this on their own and takes away the feelings of investment and accomplishment in their work. Also, mathematical models are not perfect and multiple models can provide very different results. Mathematician George Box summed this up beautifully when he said, “All models are wrong, but some are useful.” (Box and Draper 1987, p. 424).

WHEN STUDENTS ARE MODELING, THEY MUST BE MAKING GENUINE CHOICES.
The best problems involve making decisions about things that matter to the students and help them see how using mathematics can help them make good, informed decisions.

START BIG, START SMALL, JUST START.
You may feel ready to jump in and make big changes, and if so, that is great! However, even small changes to things you already do in your classroom can encourage students to engage in mathematical modeling. To start small, choose a mathematical modeling task that you feel comfortable with – maybe one that you and your colleagues tackle while engaging in the mathematical modeling framework.

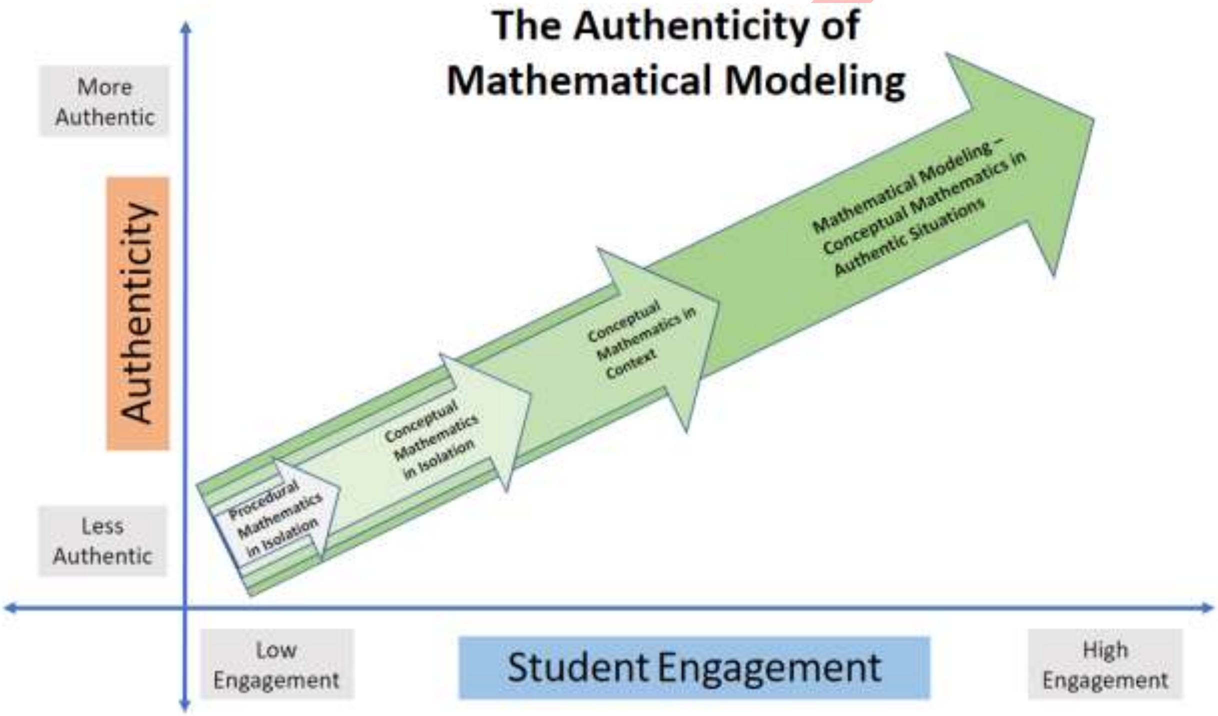
ASSESSMENT SHOULD FOCUS ON THE PROCESS, NOT THE PRODUCT.
Mathematical models (and the results they produce) are intimately tied to the assumptions made in creating the models. Assessment should be in service of helping students improve their ability to model, which will, in time, translate to a better product.

MODELING DOES NOT HAPPEN IN ISOLATION.
Whether students are working in teams, sharing ideas with the whole class, or going online to do research or collect data, modeling is not about working in a vacuum. The problems are challenging, and it helps to know you have support as you seek answers.

Adapted from GAIMME, 2016

Mathematical Modeling Continuum

The Mathematical Modeling Continuum provides a visual and applicable flow map that illustrates how we can address 21st century (cognitive, intrapersonal, interpersonal) competencies domains through a progression of the four stages of mathematical problem development: Mathematics Problems, Word Problems, Application Problems and Mathematical Modeling Problems. The eventual objective of the continuum is to develop a process in which students learn to transfer knowledge in one situation and apply it to new situations.



Framework for Statistical Reasoning

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

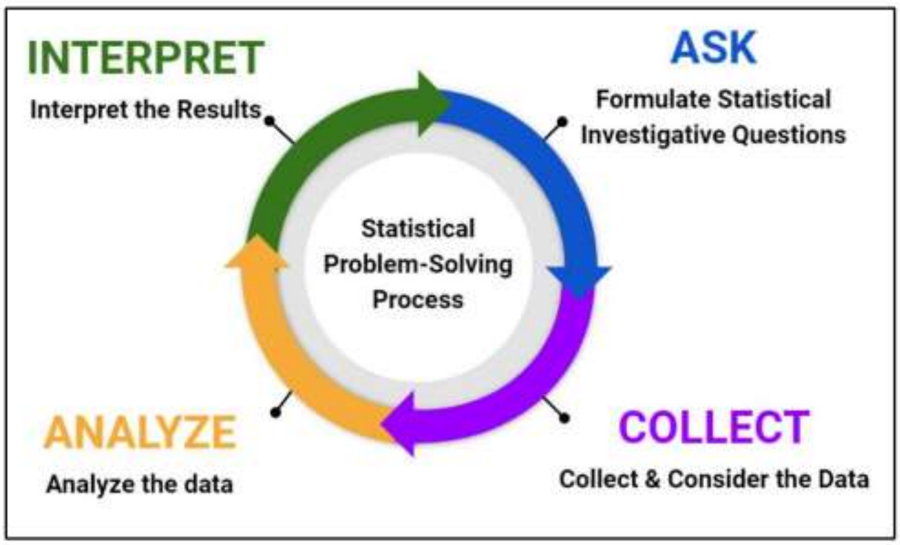


FIGURE 1: GEORGIA FRAMEWORK FOR STATISTICAL REASONING

I. **Formulate a statistical investigative question:** Oftentimes questions will naturally come about in various content areas.

II. **Collect and Consider Data:** Depending on the question asked, students will collect numerical or categorical data.

III. **Analyze the Data:** Make sense of the data and communicate what the data mean using graphical displays and words.

IV. **Interpret the Data:** Answer the original question based on the data collected. As students answer one question, others will develop. This is a great way to continue the statistical reasoning.

SECTION 6: Progressions

Vertical Learning Progressions

The K-12 Mathematics Learning Progressions document provides a visual progression of mathematics expectations within Georgia's K-12 Mathematics Standards across all levels for students, parents, and educators to make connections among key concepts as students move from one grade level and course to the next. Adequate time should be spent analyzing and understanding the progression of standards.

The progressions will:

- deepen your understanding of how the development of algebraic and geometric thinking has proven to be a critical element of a student's mathematics success as they transition from middle school to high school
- inform you of your course expectations compared to other courses in the progression
- help you plan more effectively with same-course and other course colleagues
- deepen your understanding of the mathematics you teach
- help you understand how algebraic thinking develops prior to the course, in the course, and beyond the course in order to support the students' development of algebraic thinking
- inform you of where students should be in their mathematical journey based on what they should understand from the prior years
- show you how conceptual understanding develops, making it easier to help students who have gaps in learning
- help your students see the connections between ideas in mathematics in your course and beyond by helping them connect what they already know to what is to come
- help you assess understanding more completely and develop better assessments

The progressions document can be found here: [K-12 Mathematics Learning Progressions.]

SECTION 7: Big Ideas

Big Ideas, High School

The image below illustrates the big ideas for Georgia's K-12 Mathematics Standards. For each big idea in Kindergarten through High School Advanced Algebra: Concepts & Connections, the grade levels/courses where it is addressed is indicated. Most of the big ideas span multiple grade levels/courses, supporting the progression of mathematics and the coherence across grade levels/courses. Each big idea is unpacked at the standard level.

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>HS Algebra: Concepts & Connections</th><th>HS Geometry: Concepts & Connections</th><th>HS Advanced Algebra: Concepts & Connections</th></tr>
<tr><td colspan="12">Mathematical Modeling (MM)</td></tr>
<tr><td colspan="12">Mathematical Practices (MP)</td></tr>
<tr><td colspan="12">Data & Statistical Reasoning (DSR)</td></tr>
<tr><td colspan="12">Numerical Reasoning (NR)</td></tr>
<tr><td colspan="12">Patterning & Algebraic Reasoning (PAR)</td></tr>
<tr><td colspan="12">Geometric & Spatial Reasoning (GSR)</td></tr>
<tr><td colspan="6">Measurement & Data Reasoning (MDR)</td><td colspan="6"></td></tr>
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SECTION 8: Standards Analysis

Content Standards

Georgia's K-12 Mathematics Standards are for ALL learners. All students, including students with disabilities, English Learners (EL) and students identified as gifted, must be challenged to excel within the general curriculum and be prepared for success in their post-school lives, including college, military enlistment, and/or careers.

Each student is expected to meet the high academic standards and demonstrate the level of mathematical reasoning needed to fully develop their conceptual understanding and procedural fluency; therefore, their instruction must incorporate supports and accommodations.

Promoting a culture of high expectations for all students is a fundamental goal of Georgia's K-12 Mathematics Standards.

ALGEBRA: CONCEPTS & CONNECTIONS STANDARDS
A.MP: **Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.**
A.MM.1**: Apply mathematics to real-life situations; model real-life phenomena using mathematics.**
A.FGR.2**: Construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. Use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs.**
A.GSR.3: **Solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.**
A.PAR.4**: Create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena**
A.NR.5**: Investigate rational and irrational numbers and rewrite expressions involving square roots and cube roots.**
A.PAR.6**: Build quadratic expressions and equations to represent and model real-life phenomena; solve quadratic equations in mathematically applicable situations.**
A.FGR.7**: Construct and interpret quadratic functions from data points to model and explain real-life phenomena; describe key characteristics of the graph of a quadratic function to explain a mathematically applicable situation for which the graph serves as a model.**
A.PAR.8**: Create and analyze exponential expressions and equations to represent and model real-life phenomena; solve exponential equations in mathematically applicable situations.**
A.FGR.9**: Construct and analyze the graph of an exponential function to explain a mathematically applicable situation for which the graph serves as a model; compare exponential with linear and quadratic functions.**
A.DSR.10**: Collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems; Represent**

bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.

Understanding the Content Standards

Clicking on each of the standards below will provide a brief description of the standard along with a breakdown of the standard through its learning objectives. For more detailed information about how to help students build toward mastery of these standards and background information, review *Explanation of the Mathematics Content Standards*.

Algebra: Concepts & Connections	
Mathematical Practice A.MP	
Mathematical Modeling A.MM.1	
Numerical Reasoning A.NR.5	**Patterning & Algebraic Reasoning** A.PAR.4 A.PAR.6 A.PAR.8
Functional & Graphical Reasoning A.FGR.2 A.FGR.7 A.FGR.9	**Geometric & Spatial Reasoning** A.GSR.3
Data & Statistical Reasoning A.DSR.10	

Understanding the Content Standards

MATHEMATICAL PRACTICES STANDARD/KEY COMPETENCY
MATHEMATICAL PRACTICES – *reasoning and explaining, modeling, and using tools, seeing structure, and generalizing*
A.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories. These mathematical practices describe how students should engage with the mathematics content for their grade level or course. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Mathematical Practices present.
Breakdown of Standard/Key Competency (Expectation/Learning Objective)
A.MP.1 Make sense of problems and persevere in solving them.
A.MP.2 Reason abstractly and quantitatively.
A.MP.3 Construct viable arguments and critique the reasoning of others.
A.MP.4 Model with mathematics.
A.MP.5 Use appropriate tools strategically.
A.MP.6 Attend to precision.
A.MP.7 Look for and make use of structure.
A.MP.8 Look for and express regularity in repeated reasoning.

STANDARD/KEY COMPETENCY 1
MATHEMATICAL MODELING
A.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students should be able to model with mathematics. As they engage in mathematical modeling, they develop a more engaging and deeper understanding of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed life decisions based on data and the models they create. For this reason, the modeling unit will be embedded throughout the course. Use the Mathematical Modeling Framework as an instructional support.
Breakdown of Standard/Key Competency 1 **(Expectation/Learning Objective)**
A.MM.1.1 Explain applicable, mathematical problems using a mathematical model.
A.MM.1.2: Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities domains.
A.MM.1.3: Use units of measure (linear, area, capacity, rates, and time) as a way to make sense of conceptual problems; identify, use, and record appropriate units of measure within the given framework, within data displays, and on graphs; convert units and rates using proportional reasoning given a conversion factor; use units within multi-step problems and formulas; interpret units of input and resulting units of output.
A.MM.1.4: Use various mathematical representations and structures with this information to represent and solve real-life problems.
A.MM.1.5: Define appropriate quantities for the purpose of descriptive modeling.

STANDARD/KEY COMPETENCY 2
FUNCTIONAL & GRAPHICAL REASONING – *function notation, modeling linear functions, linear vs. nonlinear comparisons*
A.FGR.2: Construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. Use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. They will use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs.
Breakdown of Standard/Key Competency 2 (Expectation/Learning Objective)
A.FGR.2.1: Use mathematically applicable situations algebraically and graphically to build and interpret arithmetic sequences as functions whose domain is a subset of the integers.
A.FGR.2.2: Construct and interpret the graph of a linear function that models real-life phenomena and represent key characteristics of the graph using formal notation.
A.FGR.2.3: Relate the domain and range of a linear function to its graph and, where applicable, to the quantitative relationship it describes. Use formal interval and set notation to describe the domain and range of linear functions.
A.FGR.2.4: Use function notation to build and evaluate linear functions for inputs in their domains and interpret statements that use function notation in terms of a mathematical framework.
A.FGR.2.5: Analyze the difference between linear functions and nonlinear functions by informally analyzing the graphs of various parent functions (linear, quadratic, exponential, absolute value, square root, and cube root parent curves).

STANDARD/KEY COMPETENCY 3
Patterning & Algebraic Reasoning – *linear inequalities and systems of linear inequalities*
A.GSR.3: Solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will apply their understanding of linear relationships to solve real-life, application problems related to slope, parallel lines, perpendicular lines, area, and perimeter. Students extend their understanding of the relationship of the slopes of parallel and perpendicular lines. Given these relationships, students find equations of lines parallel and perpendicular to a given line, as well as identify whether or not lines are parallel or perpendicular given their equations. Students calculate the area and perimeter of special parallelograms and triangles with simple, unknown side lengths. Applying their understanding of slope, students use the distance and midpoint formulas to solve real-world problems.
Breakdown of Standard/Key Competency 3 **(Expectation/Learning Objective)**
A.GSR.3.1 Solve real-life problems involving slope, parallel lines, perpendicular lines, area, and perimeter.
A. GSR.3.2 Apply the distance formula, midpoint formula, and slope of line segments to solve real-world problems.

STANDARD/KEY COMPETENCY 4
***Patterning & Algebraic Reasoning –** linear inequalities and systems of linear inequalities*
A.PAR.4: Create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena. Exploring linear inequalities which represent realistic, real-life phenomena, students rewrite the inequality in various forms, such as slope-intercept form, for graphing. Students should be given opportunities to solve linear inequalities graphically and algebraically. Students will explore the difference between solid lines and dashed lines through exploration on an interactive graph and recognize that the graph of a linear inequality in two variables is a half-plane. Students will also use technology tools to solve systems of linear inequalities graphically.
Breakdown of Standard/Key Competency 4 (Expectation/Learning Objective)
A.PAR.4.1: Create and solve linear inequalities in two variables to represent relationships between quantities including mathematically applicable situations; graph inequalities on coordinate axes with labels and scales.
A.PAR.4.2: Represent constraints of linear inequalities and interpret data points as possible or not possible.
A.PAR.4.3: Solve systems of linear inequalities by graphing, including systems representing a mathematically applicable situation.

STANDARD/KEY COMPETENCY 5
NUMERICAL REASONING - *rational and irrational numbers, square roots, and cube roots*
A.NR.5: Investigate rational and irrational numbers and rewrite expressions involving square roots and cube roots. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering of the learning objectives within each unit.* When learning this standard, students use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots and cube roots. Through the learning experiences students develop a conceptual understanding of the sums and products of rational and irrational numbers through exploration and investigation. Students will be expected to judge the reasonableness of an answer based on their understanding of rational and irrational numbers and to be able to explain why their reasoning.
Breakdown of Standard/Key Competency 5 (Expectation/Learning Objective)
A.NR.5.1: Rewrite algebraic and numeric expressions involving radicals.
A.NR.5.2: Using numerical reasoning, show and explain that the sum or product of rational numbers is rational, the sum of a rational number and an irrational number is irrational, and the product of a nonzero rational number and an irrational number is irrational.

STANDARD/KEY COMPETENCY 6
PATTERNING & ALGEBRAIC REASONING – quadratic expressions & equations
A.PAR.6: Build quadratic expressions and equations to represent and model real-life phenomena; solve quadratic equations in mathematically applicable situations. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will analyze quadratic expressions and equations. Students begin this study on quadratics with interpreting parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context. Given mathematically applicable situations which utilize formulas or expressions with multiple terms and/or factors, students will interpret the meaning of given individual terms or factors. Students will use the structure of a quadratic expression to rewrite it in different equivalent forms and to move fluently (flexibly, accurately, efficiently) between different forms of a quadratic expression (standard, vertex, and factored forms). Students will multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to produce a quadratic expression. Students will use polynomial operations. Polynomial sums, differences, and products should not exceed a maximum degree of 2. Students should be able to solve quadratic equations fluently (flexibly, accurately, efficiently) by inspection, taking square roots, factoring, completing the square, and applying the quadratic formula, as appropriate to the initial form of the equation. Additionally, students should be able to fluently transform a quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions and analyze and explain what the zeros describe in context. Students will multiply variable expressions involving the product of a monomial and a binomial and the product of two binomials to solve a quadratic equation.
Breakdown of Standard/Key Competency 6 (Expectation/Learning Objective)
A.PAR.6.1: Interpret quadratic expressions and parts of a quadratic expression that represent a quantity in terms of its context.
A.PAR.6.2: Fluently choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the expression.
A.PAR.6.3: Create and solve quadratic equations in one variable and explain the solution in the framework of applicable phenomena.
A.PAR.6.4: Represent constraints by quadratic equations and interpret data points as possible or not possible in a modeling framework.

STANDARD/KEY COMPETENCY 7
FUNCTIONAL & GRAPHICAL REASONING – quadratic functions
A.FGR.7: Construct and interpret quadratic functions from data points to model and explain real-life phenomena; describe key characteristics of the graph of a quadratic function to explain a mathematically applicable situation for which the graph serves as a model. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will apply their understanding of function notation from their work with linear functions to build, evaluate, and interpret quadratic functions using function notation. Students interpret the domain given a function expressed numerically (in tables), algebraically, graphically and as verbal descriptions and experiment with cases and illustrate an explanation of the effects on the graph using technology. Students will sketch a graph showing key features (domain, range, and intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; asymptotes; end behavior) expressed in interval and set-builder notation using inequalities. Students will interpret the maximum and minimum value of a quadratic function expressed in a variety of ways. Within mathematical contexts, students will examine a quadratic function by analyzing the zeros, extreme values, and symmetry of the graph and interpret these properties. Students should be able to move fluently (flexibly, accurately, efficiently) between the factored form, vertex form, and standard form of a quadratic function. Students will estimate the rate of change from a graph of a quadratic function and compare the rate of change of linear functions to that of the average rate of change of quadratic functions. This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals. Further work with comparison will include comparing a quadratic function to a linear function, or another quadratic function and comparing key characteristics of quadratic functions with the key characteristics of linear functions. Through multiple learning experiences with comparisons, students will observe using graphs and tables that a quantity increasing quadratically will eventually exceed a portion of a quantity increasing linearly.
Breakdown of Standard/Key Competency 7 (Expectation/Learning Objective)
A.FGR.7.1: Use function notation to build and evaluate quadratic functions for inputs in their domains and interpret statements that use function notation in terms of a given framework.
A.FGR.7.2: Identify the effect on the graph generated by a quadratic function when replacing f(x) with f(x) + k, kf(x), f(kx), and f (x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
A.FGR.7.3: Graph and analyze the key characteristics of quadratic functions.
A.FGR.7.4: Relate the domain and range of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes.
A.FGR.7.5: Rewrite a quadratic function representing a mathematically applicable situation to reveal the maximum or minimum value of the function it defines. Explain what the value describes in context.

A.FGR.7.6: Create quadratic functions in two variables to represent relationships between quantities; graph quadratic functions on the coordinate axes with labels and scales.
A.FGR.7.7: Estimate, calculate, and interpret the average rate of change of a quadratic function and make comparisons to the average rate of change of linear functions.
A.FGR.7.8: Write a function defined by a quadratic expression in different but equivalent forms to reveal and explain different properties of the function.
A.FGR.7.9: Compare characteristics of two functions each represented in a different way.

STANDARD/KEY COMPETENCY 8
PATTERNING & ALGEBRAIC REASONING – *exponential expressions and equations*
A.PAR.8: Create and analyze exponential expressions and equations to represent and model real-life phenomena; solve exponential equations in mathematically applicable situations.   **Understanding the Intent and Rigor of the Standard**   *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.*   When learning this standard, students will interpret exponential expressions, investigate and solve one variable exponential equations in context, and understand parameters of two variable exponential equations. Students will interpret parts of an expression, such as terms, factors, leading coefficient, coefficients, constant and degree in context. Given mathematically applicable situations which utilize formulas or expressions with multiple terms and/or factors, students should be able to interpret the meaning in context of individual terms or factors.   Students look for patterns in contextual situations and other numeric patterns and create equations from them. The equations can arise from many situations. Exponential equations are limited to those containing like bases, or exponential equations that could easily be transferred to like bases with linear operations.
Breakdown of Standard/Key Competency 8 (Expectation/Learning Objective)
A.PAR.8.1: Interpret exponential expressions and parts of an exponential expression that represent a quantity in terms of its framework.
A.PAR.8.2: Create exponential equations in one variable and use them to solve problems, including mathematically applicable situations.
A.PAR.8.3: Create exponential equations in two variables to represent relationships between quantities, including in mathematically applicable situations; graph equations on coordinate axes with labels and scales.
A.PAR.8.4: Represent constraints by exponential equations and interpret data points as possible or not possible in a modeling environment.

STANDARD/KEY COMPETENCY 9
FUNCTIONAL & GRAPHICAL REASONING – *exponential functions*
A.FGR.9: Construct and analyze the graph of an exponential function to explain a mathematically applicable situation for which the graph serves as a model; compare exponential with linear and quadratic functions. **Understanding the Intent and Rigor of the Standard** *This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.* When learning this standard, students will apply their understanding of function notation from their work with linear and quadratic functions to build, evaluate, and interpret exponential functions using function notation. Students will interpret the domain given a function expressed numerically, algebraically, and graphically. Students will experiment with cases and illustrate an explanation of the effects on the graph using interactive technology. Students will make connections between exponential functions and geometric sequences. Sequences can be defined recursively and explicitly. Further work with geometric sequences focuses on building and interpreting geometric sequences and covert geometric sequences from explicit form to recursive and vice versa. Students should have ample opportunities to compare geometric sequences with arithmetic sequences presented in a variety of ways. Students will compare an exponential function to a linear function, a quadratic function, or to another exponential function. Additionally, students compare key characteristics of exponential functions with the key characteristics of linear and quadratic functions. From their study, students should be able to observe using graphs and tables that a quantity increasing quadratically will eventually exceed a portion of a quantity increasing linearly and a quantity increasing exponentially will eventually exceed a portion of a quantity increasing linearly or quadratically.
Breakdown of Standard/Key Competency 9 **(Expectation/Learning Objective)**
A.FGR.9.1: Use function notation to build and evaluate exponential functions for inputs in their domains and interpret statements that use function notation in terms of a context.
A.FGR.9.2: Graph and analyze the key characteristics of simple exponential functions based on mathematically applicable situations.
A.FGR.9.3: Identify the effect on the graph generated by an exponential function when replacing f(x) with f(x) + k, and k f(x), for specific values of k (both positive and negative); find the value of k given the graphs.
A.FGR.9.4: Use mathematically applicable situations algebraically and graphically to build and interpret geometric sequences as functions whose domain is a subset of the integers.
A.FGR.9.5: Compare characteristics of two functions each represented in a different way.

STANDARD/KEY COMPETENCY 10

DATA & STATISTICAL REASONING – *univariate data and single quantitative variables; bivariate data*

A.DSR.10: Collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems; Represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.

Understanding the Intent and Rigor of the Standard

This standard consists of a breakdown through several learning objectives. These learning objectives are not meant to be taught in isolation, but rather in clusters of related learning objectives. The Algebra: Concepts & Connections curriculum map provides suggestions for clustering learning objectives within each unit.

When learning this standard, students will collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems. Students will represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems. Students were first introduced to the concept of MAD as a tool for comparing variability of multiple data sets in sixth grade mathematics; students use the meaning of mean absolute deviation (MAD) to interpret the meaning of standard deviation.

Students will describe the direction, strength, and form (linear, non-linear) of the association between two quantitative variables and utilize interactive graphing technologies to model linear data and make sense of the slope (predicted rate of change) visually. Students should also utilize interactive graphing technologies to interpret the correlation coefficient, r. Students will use the line of best fit and the correlation coefficient, r, to make predictions and describe the reasonableness of the prediction in the investigation of a practical, real-life situation.

Breakdown of Standard/Key Competency 10 (Expectation/Learning Objective)

A.DSR.10.1: Use statistics appropriate to the shape of the data distribution to compare and represent center (median and mean) and variability (interquartile range, standard deviation) of two or more distributions by hand and using technology.

A.DSR.10.2: Interpret differences in shape, center, and variability of the distributions based on the investigation, accounting for possible effects of extreme data points (outliers).

A.DSR.10.3: Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

A.DSR.10.4: Interpret the slope (predicted rate of change) and the intercept (constant term) of a linear model based on the investigation of the data.

A.DSR.10.5: Calculate the line of best fit and interpret the correlation coefficient, r, of a linear fit using technology. Use r to describe the strength of the goodness of fit of the regression. Use the linear function to make predictions and assess how reasonable the prediction is in context.

A.DSR.10.6: Decide which type of function is most appropriate by observing graphed data.

A.DSR.10.7: Distinguish between correlation and causation.

SECTION 9: Instructional Supports

Curriculum Units

SEMESTER 1

Unit 1: Modeling Linear Functions	3 - 4 Weeks
Students will construct and interpret arithmetic sequences as functions, algebraically and graphically, to model and explain real-life phenomena. They will use formal notation to represent linear functions and the key characteristics of graphs of linear functions, and informally compare linear and non-linear functions using parent graphs.	
A.FGR.2, A.MM.1, A.MP. 1-8	

Unit 2: Analyzing Linear Inequalities	5 – 6 weeks
Students will create, analyze, and solve linear inequalities in two variables and systems of linear inequalities to model real-life phenomena.	
A.PAR.4, A.MM.1, A.MP.1-8	

Unit 3: Investigating Rational and Irrational Numbers	1 – 2 weeks
Students will investigate rational and irrational numbers and rewrite expressions involving square roots and cube roots. They should be able to use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots and cube roots.	
A.NR.5, A.MP.1-8	

Unit 4: Modeling and Analyzing Quadratic Functions	6 – 7 weeks
Students will analyze quadratic functions. Students will (1) investigate key features of graphs; (2) solve quadratic equations by taking square roots, factoring (x^2 + bx + c AND ax^2 + bx + c), completing the square, and using the quadratic formula; (3) compare and contrast graphs in standard, vertex, and intercept forms. Students will only work with real number solutions.	
A.PAR.6, A.FGR.7, A.MM.1, A.MP.1-8	

Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

SEMESTER 2

Unit 5: Modeling and Analyzing Exponential Expressions and Equations	2 – 3 weeks
Students will interpret exponential expressions, one variable exponential equations in context, and understand parameters of two variable exponential equations	
	A.PAR.8, A.MM.1, A.MP.1-8

Unit 6: Analyzing Exponential Functions	4 – 5 weeks
Students will construct and analyze the graph of an exponential function to explain a contextual situation for which the graph serves as a model; compare exponential with linear and quadratic functions.	
	A.FGR.9, A.MM.1, A.MP.1-8

Unit 7: Investigating Data	3 – 4 weeks
Students will collect, analyze, and interpret univariate quantitative data to answer statistical investigative questions that compare groups to solve real-life problems. Students will represent bivariate data on a scatter plot and fit a function to the data to answer statistical questions and solve real-life problems.	
	A.DSR.10, A.MM.1, A.MP.1-8

Unit 8: Algebraic Connections to Geometric Concepts	2 – 3 weeks
Students will solve problems involving distance, midpoint, slope, area, and perimeter to model and explain real-life phenomena.	
	A.GSR.3, A.MM.1, A.MP.1-8

Unit 9: Culminating Capstone Unit	1 - 2 weeks
(applying concepts in real-life contexts in a culminating interdisciplinary unit) The capstone unit applies content learned in previous interdisciplinary PBLs and units throughout the school year. The capstone unit is an interdisciplinary unit that allows students to create a presentation, report, or demonstration that could include their models used to answer an overarching driving question. (e.g., Students can present their solution(s), findings, project, or answer to the driving question to a larger audience during the culminating capstone unit.)	
	ALL STANDARDS AND LEARNING OBJECTIVES

Mathematical Practices (A.MP.1- 8) should be evidenced at some point throughout each unit depending on the tasks that are explored. It is important to note that MPs 1, 3 and 6 should support the learning in every lesson.

Instructional Design

The learning plans within the units follow a modified 5-E model. The components of this model include diagnostic assessments, Engage, Explore, Apply, and Reflect sections. Also included are sections focusing on Evidence of Student Success, Student Learning Supports, and Engaging Families. Below is a list of each component included in the instructional design along with a brief description.

- **Diagnostic Assessment –** A brief diagnostic assessment for the standard is included in each learning plan. One diagnostic may be used for multiple tasks that address a specific standard.
- **Engage -** Within this section, the learning experiences include evidence-based instructional strategies that can be used as an introduction that mentally engages students to capture their interest, provides an opportunity to communicate what they know, and allows them to connect what they know to new ideas.
- **Explore -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to engage in hands-on activities to explore the new concept/big idea at a deep level.
- **Apply -** Within this section, the learning experiences include evidence-based instructional strategies that allow students to apply what they have learned in a new situation to develop a deeper understanding of the big idea.
- **Reflect -** Within this section, the learning experiences include evidence-based instructional strategies that allow students the opportunity to review and reflect on their own learning and new understandings.
- **Evidence of Student Success –** Within this section, formative assessment questions have been provided as a tool for teachers to determine the level of student understanding of the standard.
- **Student Learning Supports –** Within this section, suggested strategies are outlined to support learners as they progress towards instructional goals. Teachers should use frequent formative assessment information to determine which students need additional support. For more information on supporting the learning, extending the learning and language supports, please review the information under Instructional Support Strategies within the Comprehensive Course Overview.
- **Engaging Families –** Within this section, activities are included for families to use to build on mathematical ideas and continue to extend mathematics learning outside of school.

Mathematical Instructional Tools

Use of Manipulatives

Students can use concrete manipulatives and virtual simulations to develop the mathematical ideas that are presented in the standards in order to create a basis for the abstract reasoning necessary.

Concrete materials help students make sense of mathematical representations. Mathematical representations include the use of mathematical symbols to represent the concrete mathematical idea, thought process, or situation. This is a very important, yet often neglected, step along the way. Mathematical representations can be concrete, representational, and abstract. Each type of representation is important to student understanding. Mathematizing a situation or problem is also necessary to build understanding. Mathematizing means to take any situation and view it through the lens of mathematics.

The following are simple rules of thumb for using manipulatives:

- Introduce new manipulatives/tools by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available manipulatives/tools to use in solving problems.
- Encourage the use of a manipulative or tool at the beginning of student development of a concept and at any point when you believe it would be helpful to a student to conceptualize the mathematics presented (Van de Walle and Lovin, Teaching Student-Centered Mathematics).

Concrete-Representational-Abstract (CRA) Instructional Approach

Concrete-Representational-Abstract (CRA) is a systematic and sequential Research-Based Educational approach to teaching and learning mathematics. With CRA, students are introduced to mathematics concepts using manipulatives or hands-on materials (concrete), pictorial representations or "seeing" stage involves using images, charts, and graphs to represent objects (representational), and, lastly, abstract figures, or "symbolic' stage such as numbers and mathematical symbols that can be used to represent and solve contextual problems mathematically. Teachers help students bridge the connection between the concrete, representational, and abstract representations of the mathematics used to solve the problems.

Why is CRA so important? All students learn differently. Some students may need very little time with concrete objects or representations before they can jump to the abstract. Other students need the visuals for a longer period to make sense of mathematical ideas. We have a variety of learning styles in our classrooms, so varying the way that we approach teaching mathematics with the students is helpful in that respect. In addition, students at each stage of this progression can deepen their understanding of the concepts being learned by sharing their learning experiences during mathematics lessons.

CRA is also connected to mathematical modeling. Mathematical Modeling means to "mathematize" a situation or problem, to take a situation which might, at first glance, not seem mathematical, and view it through the lens of mathematics. Modeling may include the use of mathematical symbols, in addition to concrete materials, to represent the situation or problem. Mathematical modeling can be concrete, representational, and abstract and each type of model is important to student understanding.

Almost all topics in mathematics can be taught using CRA. However, students do not have to progress through the concrete to get to the representational and abstract stages. Students often work at the concrete and abstract or representational and abstract stages simultaneously. The reason for engaging students in this progression is to help them make sense of mathematics.

Evidence of Student Success

The value of formative assessments and feedback is tremendous. Continuous progress monitoring with both feedback and commentary should occur throughout instructional lessons.

Summative Assessments

A summative assessment is an evaluation tool generally used at the end of an assignment, unit, project, or at the end of the course. Evaluative criteria should be incorporated to assess student learning. In an educational setting, summative assessments tend to be more formal kinds of assessments (e.g., unit tests, final exams, projects, reports, and state assessments) and are typically used to assign students a course grade or to certify student mastery of intended learning outcomes for the Georgia Mathematics Standards.

Guiding Questions:

- What are the evaluative criteria (or rubric) and how do they measure student proficiency for your objectives?
- Are the assessments aligned with approved standards and learning targets?

Formative Assessments

A formative assessment is an evaluation tool used to guide and monitor the progress of student learning during instruction. Formative assessments should align to the rigor of Georgia's K-12 Mathematics Standards and the corresponding summative assessment. Its purpose is to provide continuous feedback to both the student and the teacher concerning learning successes and failures. Formative assessments diagnose skill and knowledge gaps, measure progress, and evaluate instruction. Teachers use formative assessments to determine what concepts require more teaching and what teaching techniques require modification. Educators use results of these assessments to improve student performance. Formative assessments would not necessarily be used for grading purposes. Examples include (but are not limited to): pre/posttests, portfolios, benchmark assessments, quizzes, teacher observations, teacher/student conferencing, teacher commentary and feedback. Here are some guiding questions you should consider when developing formative assessments in your classroom.

Guiding Questions:

- How will students demonstrate their understanding?
- Why should there be more than one form of assessment for students?
- In what ways will student learning be monitored during the lesson and how will this guide your instruction?
- How will feedback support students meeting the goals of the lesson?

Learning Targets and Success Criteria

Create Learning Targets: Learning targets frame a lesson from the student point of view. A learning target helps students grasp the lesson's purpose-- why it is crucial to learn this chunk of information, on this day, and in this way. Learning targets written in a student friendly way are often posted beginning with the words "I CAN..." Learning targets should clearly state what you expect students to know, understand and/or be able to do at the end of the lesson. This is called the “Learning Intention.”

Learning targets are written using observable, measurable actions and should align with the content standards identified. This component is called the **“Success Criteria.”**

Types of Learning Targets:

1. Content Knowledge
2. Strategy Development
3. Thinking/Reasoning Development
4. Procedural
5. Investigative or Inquiry
6. Reflective
7. Skills
8. Product

Guiding Questions:

- As a result of the lesson, what should students know and be able to do?
- Why is it important that students achieve this new learning – what will they be able to do as a result of this new learning?
- How is the learning target meaningful and relevant beyond the specific task/activity? Does it relate to the content standards?
- Is the task or activity aligned with the learning target?

For a detailed description of each element of the Evidence of Student Learning section, please refer to the Division of School and District Effectiveness for the Georgia Department of Education System for Effective School Instruction .

Standards-Based Grading

- Grading practices should emphasize mastery of standards through the frequent use of aligned quizzes and tests, both formative and summative.
- Continual progress monitoring should be used to assess and diagnose each student’s strengths and weaknesses based on the standards of the associated core academic mathematics course and to provide appropriate interventions.
- Opportunities should be provided for students to review content with a focus on standards not previously mastered.

Suggested Assessment Ideas Choice Board

Formative Assessments		Task-Based Performance Assessments	Classroom Challenges
• Ticket Out the Door • My Favorite No • 3-2-1 • Think-Pair-Share • Jigsaw	• Journals • Gallery Walk • Frayer Model • K-W-L Chart	• 3-Act Tasks • Open Middle • Pattern Talks • Number Talks	• Index of Classroom Challenges

Student Learning Supports

Fluency and Numeracy Development

Fluency involves three ideas: accuracy (attending to precision), efficiency (using well-understood strategies with ease), and flexibility (using flexible thinking to solve mathematical problems).

According to NCTM, fluency is also the ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop fluency, students need experience in integrating concepts and strategies and building on familiar strategies as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through strategic practice. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014).

Conceptual Understanding

Conceptual understanding involves three components: comprehension of mathematical concepts, operations, and relations. Students with a conceptual understanding will connect what they know about a mathematical idea to make sense of new mathematical ideas.

From Van de Walle (2016), “Conceptual understanding is a flexible web of connections and relationships within and between ideas, interpretations, and images of mathematical concepts - a relational understanding” (pg. 24).

Georgia Numeracy Project Resources

Georgia Secondary Numeracy Project

The Georgia Secondary Numeracy Project is a numeracy development resource provided by the Georgia Department of Education, which introduces teachers and teacher leaders to the trajectory by which learners acquire and build a solid foundation in numeracy which ultimately supports algebra readiness.

The Georgia Secondary Numeracy Project consists of a two-part universal screener:

- Part 1 of the universal screener is the Diagnostic Assessment. This assessment assesses students' strategy proficiency across 3 domains, Addition and subtraction, multiplication and division, and proportions and ratios. Building upon the strategy stages from the Georgia Early Numeracy Project, the Georgia Secondary Numeracy Project follows this progression.
- The second component of the universal screener is the Written Assessment. Using the Overall Strategy Stage from the Diagnostic Interview, students begin the Written Assessment on the Part equivalent to the Overall Strategy Stage from the Diagnostic Interview. The domains assessed on the Written Assessment are: Relational & Functional Reasoning, Patterning & Algebraic Reasoning, Statistical & Probability Reasoning and Geometric, Spatial & Measurement Reasoning.

In alignment with Georgia's Tiered Supports for Students, the Georgia Secondary Numeracy Project provides tools to target specific skills and provide tiered supports and interventions. To address the identified skills, the Georgia Secondary Numeracy provides numeracy development intervention tasks and activities. Within each unit in the course, intervention tables are provided to identify specific tasks and activities aligned to the standards and learning objectives discussed within the unit.

The Georgia Secondary Numeracy Project is the perfect complement to Georgia's K-12 Mathematics Standards, aligning with the multiple big ideas and aiding in the development of mathematical reasoning. The Georgia Secondary Numeracy Project supports the various mathematical concepts in Grade 8 through Advanced Algebra: Concepts & Connections.

College and Career Readiness

The GaDOE Roadmap for Reimaging K-12 Education provides teachers and leaders the opportunity to reimagine the learning experiences for students to ensure they are prepared for the future. The goal is to match their passions, their interests, and their abilities with the opportunities they are provided in school and beyond. All students have a uniqueness that can be nurtured and fostered, which can grow into something that is amazing. The goal of the Georgia Department of Education is to ensure that all students have any door of opportunity they desire open for them. Students begin exploring careers and preparing for post-secondary opportunities as early as Kindergarten. As students matriculate through elementary school and middle school, their exploratory opportunities for careers become more refined and focused to assist them with decision-making related to course taking options at the secondary level. Helping students refine their college and career interests, coupled with their strengths and goals will assist them with a great start in identifying post-secondary opportunities. Also, specific course pathways can be selected to ensure they build the foundational knowledge necessary for their college and career pathway of interest.

Providing opportunities for internships and collaboration with local business and community partners will also help students make connections between what they are learning in mathematics class with careers that use the mathematics can be very powerful and eye-opening for the learners. These opportunities should be provided to help students on their quest to become prepared for their future, they are being prepared for life.

At the secondary level, students have the flexibility to participate in personalized mathematics pathways that allow them to deeply explore advanced mathematics content in data science, statistics, quantitative reasoning, and/or calculus based on their future goals and interests. As students participate in advanced-level coursework in high school through the personalized pathways, opportunities should be provided for them to see the connections and experience the mathematics content in the real-world.

Interdisciplinary Teaching and Learning

Mathematical modeling can be explored through interdisciplinary teaching and learning. One instructional model for interdisciplinary teaching and learning is project-based learning. There are six powerful project-based instructional practices:

1. Plan Authentic, Intellectually Demanding Project-Based Learning Units Where Students Master Significant Content and Skills
2. Utilize Sustained, In-Depth Inquiry
3. Engage Students in a Collaborative Problem-Solving/Design Process
4. Foster a Classroom Environment That Supports Student Ownership of Learning
5. Engage in Ongoing and Purposeful Feedback, Revision, and Reflection
6. Include Community Partners in Project Planning, Implementation, and Reflection

Process thinking is also an important element of interdisciplinary teaching and learning. Process thinking has six components:

1. **Intellectual Challenge and Accomplishment** – this component focuses on developing students' capacity to learn deeply, think critically, and strive for excellence.
2. **Authenticity** – this component focuses on students working on projects that are meaningful and relevant to their community, their lives, and their future.
3. **Public Product** – students' work is publicly displayed, discussed, and critiqued.
4. **Collaboration** – this component focuses on building students' capacity to collaborate with other students in person or online and/or receive guidance from adult mentors and experts.
5. **Project Management –** students use a project management process that enables them to proceed effectively from project initiation to completion.
6. **Reflection –** students reflect on their work and their learning throughout the project.

The six components of process-based thinking connect nicely to the 8 Mathematical Practices that are embedded in all instructional units. These are a part of the Essential Instructional Guidance included for all grade levels and high school courses.

Instructional Support Strategies

Within each learning plan in the instructional units, specific instructional support strategies are provided in the Student Learning Supports section. These strategies are largely organized into three categories: Supporting the Learning, Extending the Learning, and Language Supports. Descriptions and rationales for these categories are written below; the actual strategies for support can be found within each specific learning plan.

Supporting the Learning
Teachers greatly influence how students perceive and approach struggle in the mathematics classroom. All students can learn to value perseverance as an expected and natural part of learning, (Principles to Actions, 2014). To close gaps in mathematical understanding, a focus should be placed on the structure and teaching strategies implemented in classrooms. Within the learning plans in each unit, supports designated as Supporting the Learning within the Student Learning Support sections will include, but are not limited to: • intervention activities specific to the learning experiences within the learning plan. • teacher actions from the High School Mathematics Strategies Toolkit tailored to the learning experiences within the learning plan.
Extending the Learning
According to Van de Walle, there are four basic strategies for adapting mathematics concepts for students who consistently demonstrate a solid understanding of the concepts of study. • *Acceleration* is characterized by self-paced learning and frequent exploration of similar topics but include higher-level thinking, more complex or abstract ideas and deeper levels of understanding or content. • *Enrichment* activities go beyond the topic of study to content that is not specifically a part of the course curriculum but is aligned with the lesson goals. • *Sophistication* provides a natural world view of mathematics when the level of complexity is increased or more depth is pursued. • *Novelty* introduces completely different materials from the regular curriculum. Within the learning plans in each unit, supports designated as Extending the Learning within the Student Learning Support sections will include, but are not limited to: • extension activities specific to the learning experiences within the task.

- instructional strategies that support students who are labeled gifted or demonstrated a solid understanding of the mathematical concepts within the learning experiences using the Enhancements from the GaDOE Talent Development Team.

Language Supports

Teachers support student's language development in the context of mathematical sense-making through meaningful "reciprocal" interactions and discourse with others. As ELs explore and connect new math concepts, they will need many well-supported opportunities to use language in listening, speaking, reading, and writing (Baker et al., 2014).

Within the learning plans in each unit, supports designated as Language Supports within the Student Learning Support sections will include, but are not limited to:

- teacher actions from the English Language Proficiency for English (as a 2nd language) Learners section of the HS-Georgia-Mathematics-Strategies-Toolkit.pdf (gadoe.org) tailored to the learning experiences within the learning plan.
- Evidence-Based Instructional Strategies: collection of vetted GA Mathematics resources and evidence – based instructional strategies that highlight the benefits of hands-on, relevant experiences in Mathematics that support multilingual learners.
- strategies and resources included in the document Mathematics Resources to Support English Learners found in the GaDOE mathematics resources provide specific evidence-based practices that indicate the benefits of hands-on, relevant learning experiences in the mathematics classroom.

SECTION 10: Additional Resources

RESOURCES FOR PROBLEM-BASED LEARNING

- ✓ Dan Meyer's Website
  Dan Meyer has created many problem-based learning tasks. The tasks have great hooks for the students and are aligned to the standards in this spreadsheet.

- ✓ Andrew Stadel
  Andrew Stadel has created many problem-based learning tasks using the same format as Dan Meyer.

- ✓ Robert Kaplinsky
  Robert Kaplinsky has created many tasks that engage students with real life situations.

TECHNOLOGY

- ✓ Desmos Activities & Explorations
- ✓ Geogebra Activities & Explorations
- ✓ Mathematics Learning Center

PROFESSIONAL LEARNING VIDEOS

- ✓ A.MM.1
- ✓ A.FGR.2
- ✓ A.GSR.3
- ✓ A.PAR.4
- ✓ A.NR.5
- ✓ A.PAR.6
- ✓ A.FGR.7
- ✓ A.PAR.8
- ✓ A.FGR.9
- ✓ A.DSR.10

GENERAL RESOURCES

- ✓ Georgia Home Classroom Digital Learning Plans
- ✓ Georgia Mathematics Strategies Toolkit to Address Learner Variability
- ✓ Algebra Guide for Effective Mathematics Instruction